Topics:

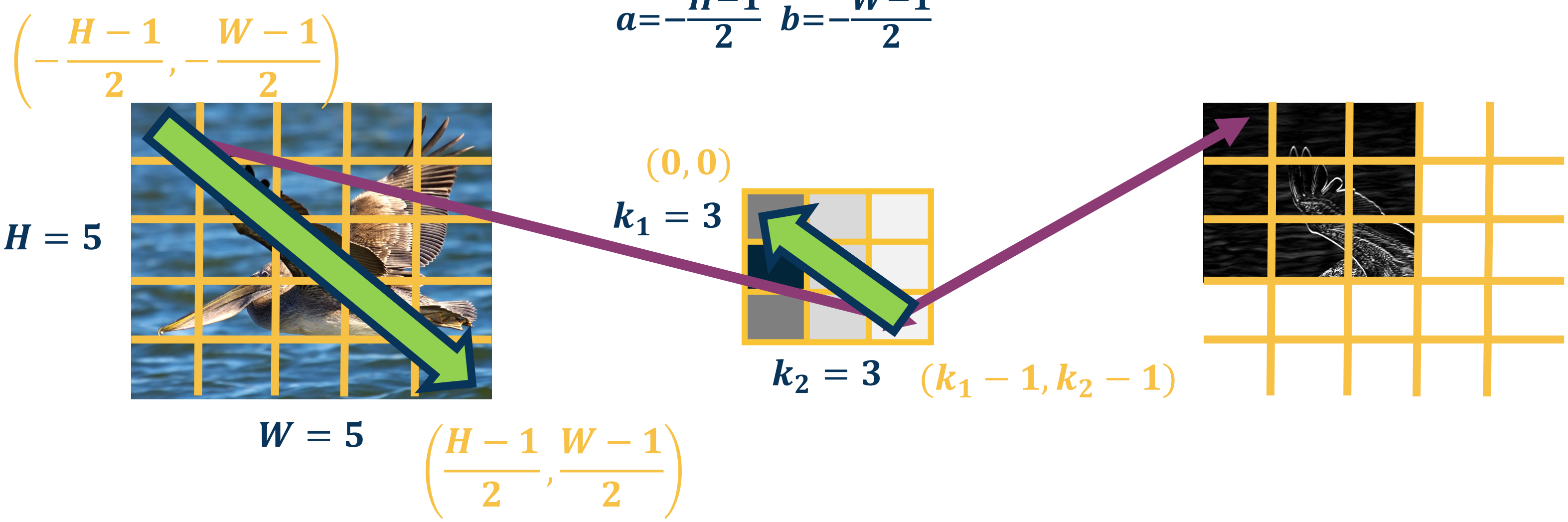
- Convolutional Neural Networks

CS 4644-DL / 7643-A
ZSOLT KIRA

- **Assignment 2**
  - Implement convolutional neural networks
  - Resources (in addition to lectures):
    - DL book: Convolutional Networks
    - CNN notes https://www.cc.gatech.edu/classes/AY2022/cs7643_spring/assets/L10_cnns_notes.pdf
    - Backprop notes https://www.cc.gatech.edu/classes/AY2023/cs7643_spring/assets/L10_cnns_backprop_notes.pdf
    - **HW2 Tutorial @190, Conv backward @192, OMSCS versions @191**
    - Slower OMSCS lectures on dropbox: Module 2 Lessons 5-6 (M2L5/M2L6) (https://www.dropbox.com/sh/iviro188gq0b4vs/AADdHxX_Uy1TkpF_yvIzX0nPa?dl=0)

- **FB/Meta Office hours Friday 02/17 2pm EST!**
  - Pytorch & scalable training
  - Module 2, Lesson 8 (M2L8), on dropbox

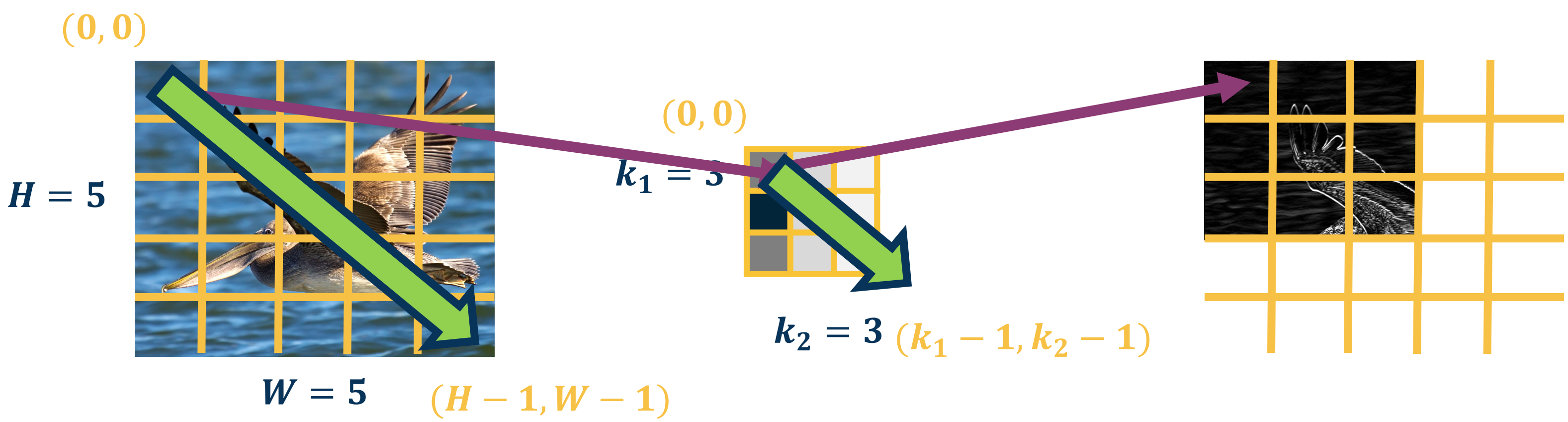
$$y(r,c) = (x * k)(r,c) = \sum_{a=-\frac{H-1}{2}}^{\frac{H-1}{2},} \sum_{b=-\frac{W-1}{2}}^{\frac{W-1}{2},} x(a,b)\, k(r-a, c-b)$$



$$y(0,0) = x(-2,-2)k(2,2) + x(-2,-1)k(2,1) + x(-2,0)k(2,0) + x(-2,1)k(2,-1) + x(-2,2)k(2,-2) + \dots$$

Mathematics of Discrete 2D Convolution

$$y(r,c) = (x * k)(r,c) = \sum_{a=0}^{k_1-1} \sum_{b=0}^{k_2-1} x(r+a, c+b)\, k(a,b)$$



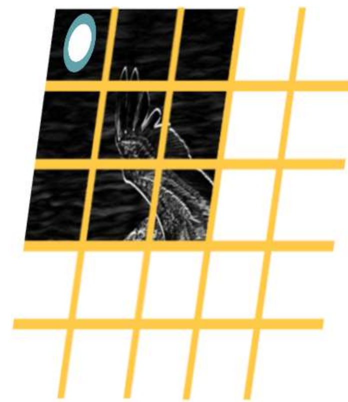
Since we will be learning these kernels, this change does not matter!

Cross-Correlation

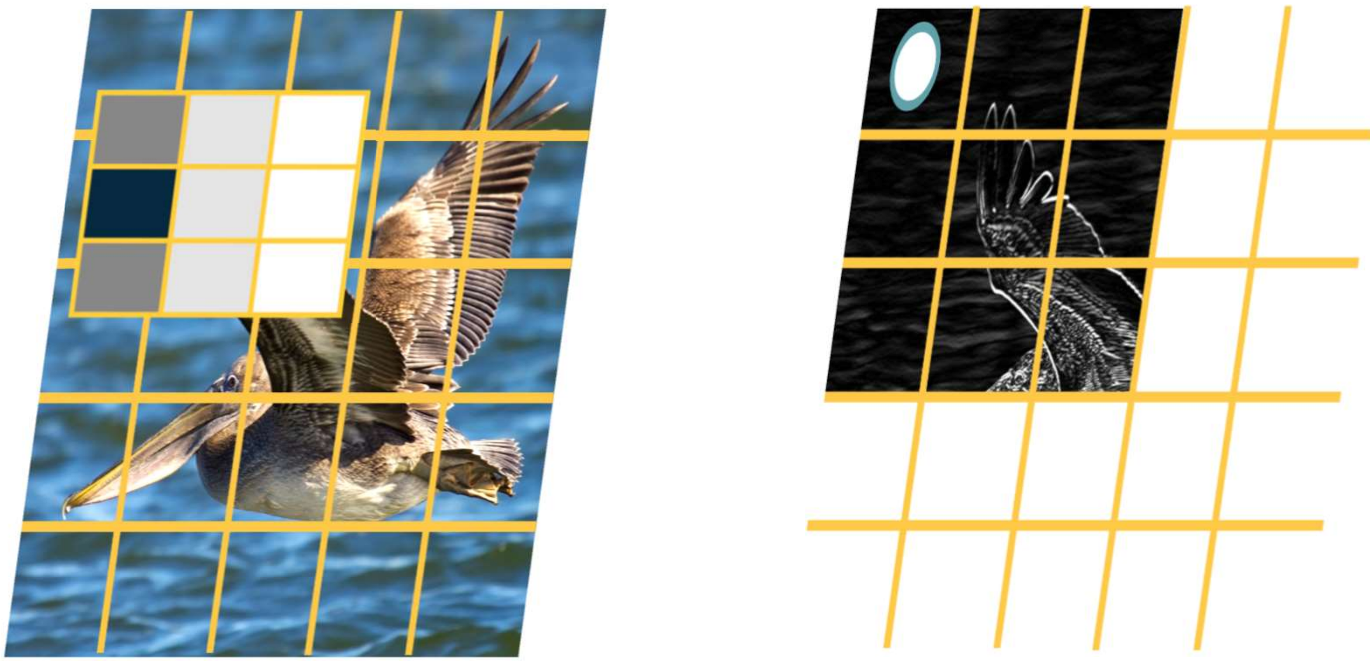
$$X(0:2,0:2) = \begin{bmatrix} 200 & 150 & 150 \\ 100 & 50 & 100 \\ 25 & 25 & 10 \end{bmatrix} \qquad K' = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$X(0{:}2,0{:}2) \cdot K' = 65 + \text{bias}$$

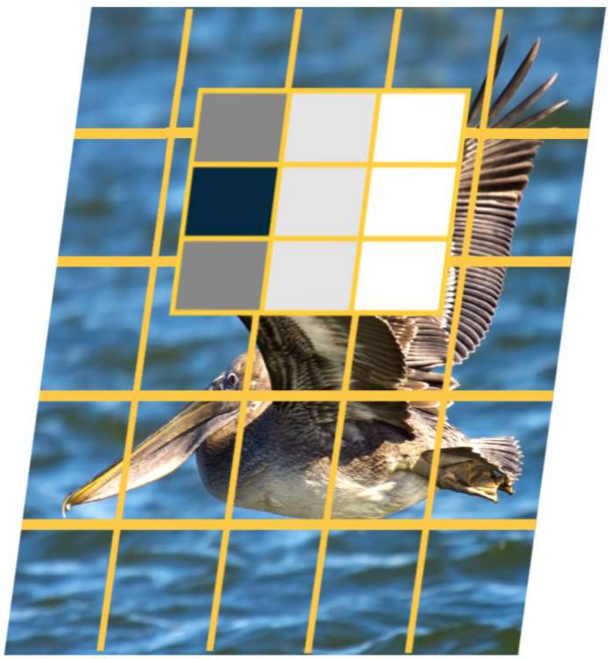
Dot product
(element-wise multiply and sum)

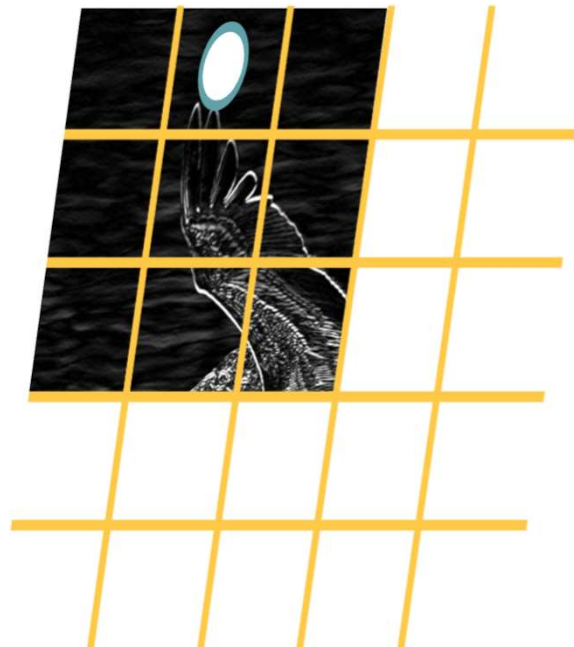


Cross-Correlation

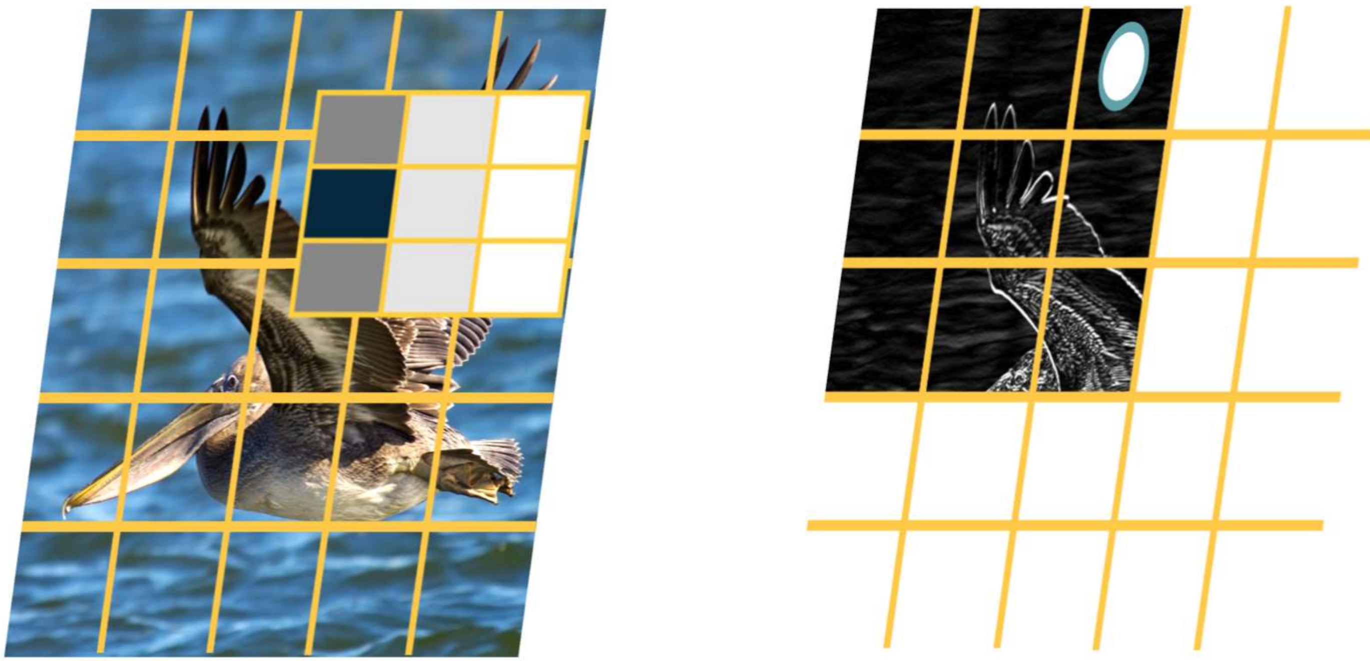


Convolution and Cross-Correlation

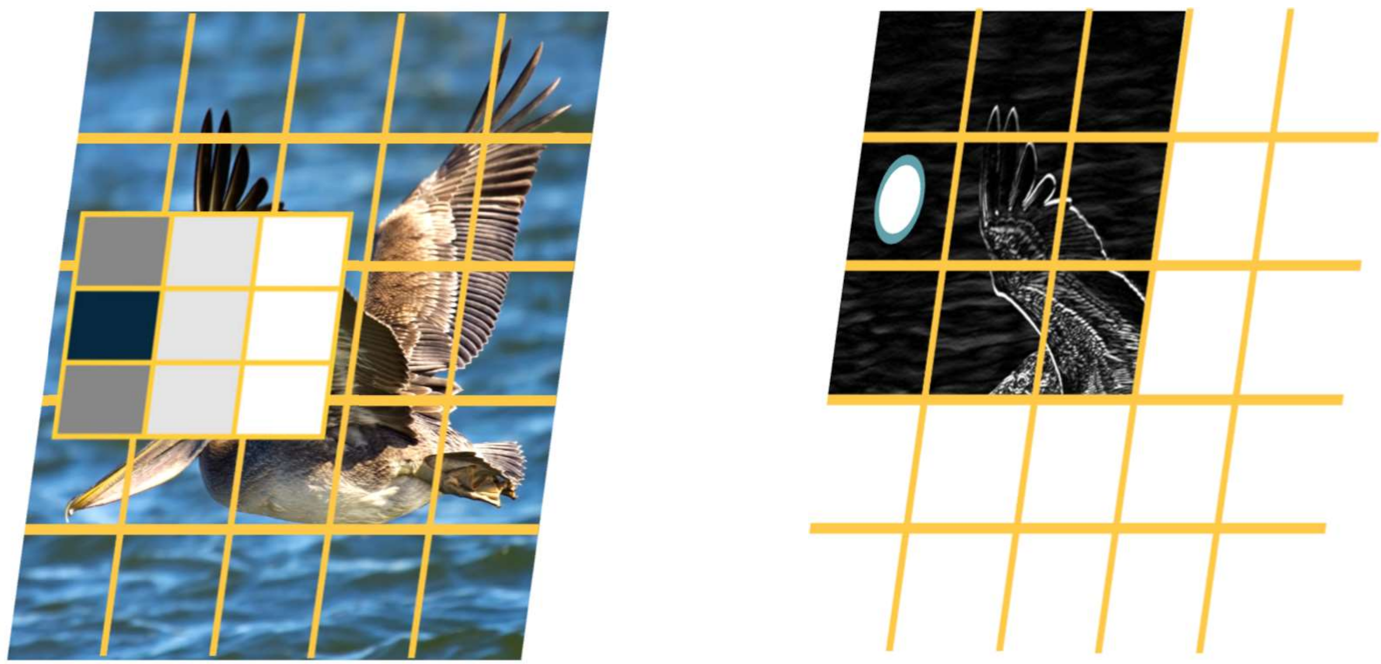




Convolution and Cross-Correlation



Convolution and Cross-Correlation



Convolution and Cross-Correlation

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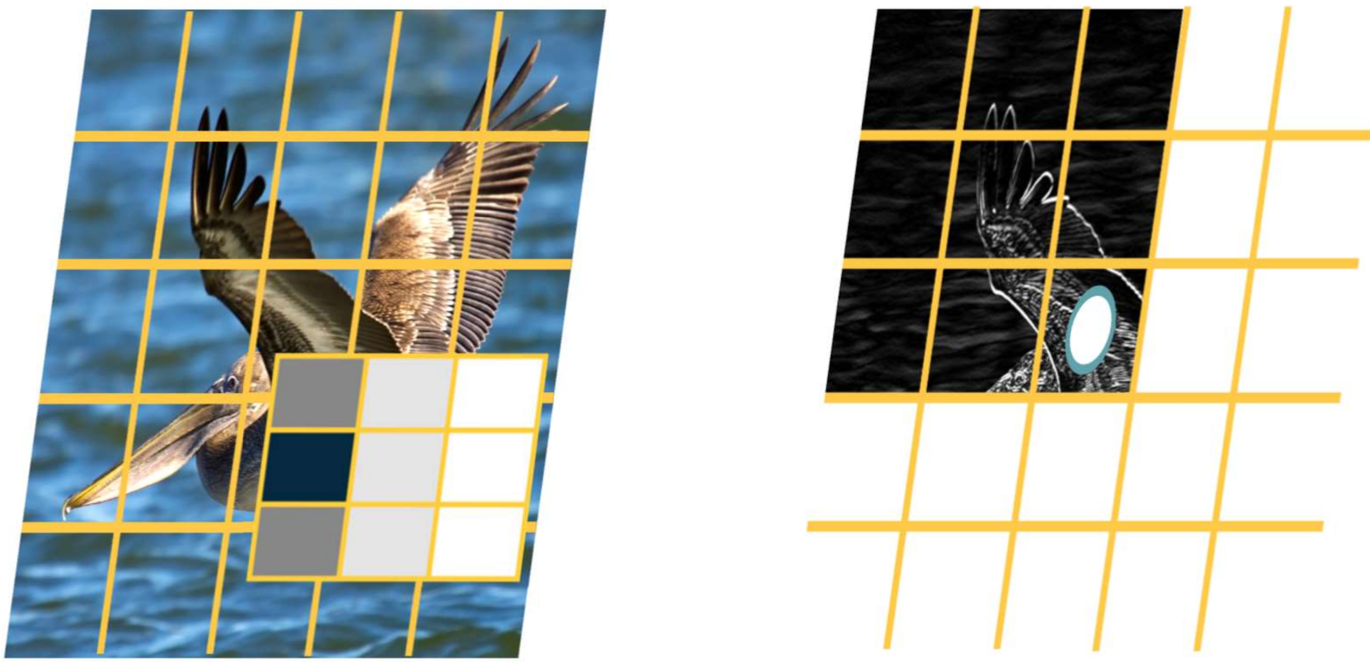
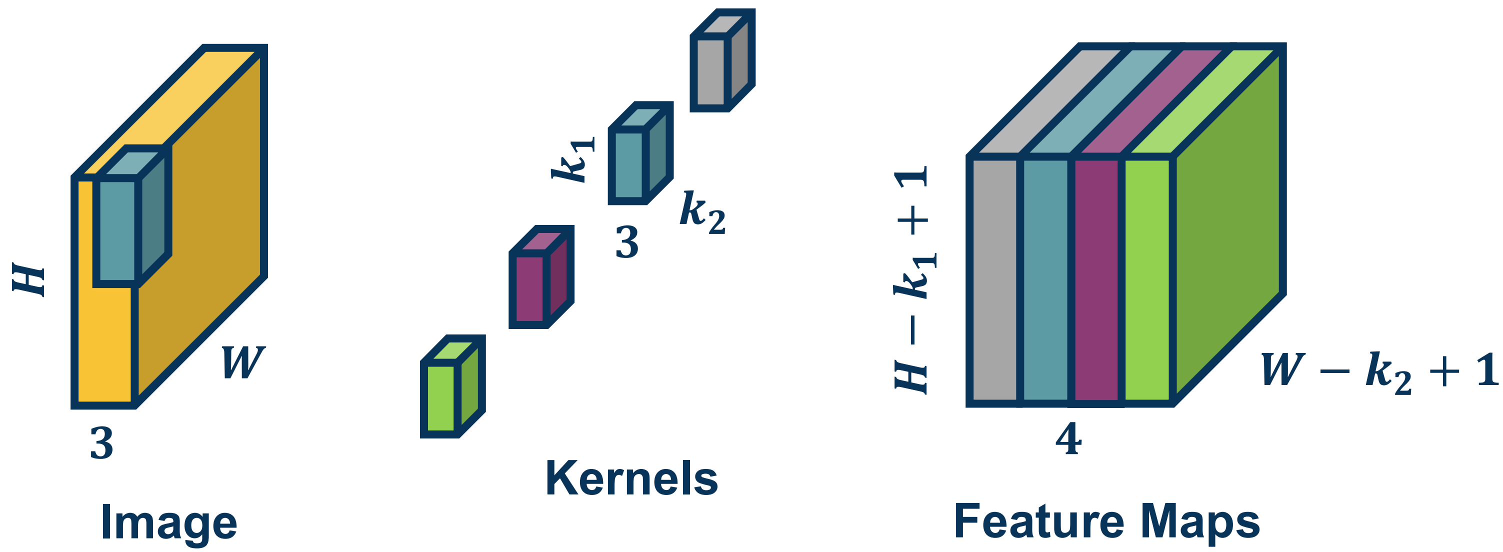


Convolution and Cross-Correlation

Number of parameters with N filters is: $N * (k_1 * k_2 * 3 + 1)$

- Example:
  $k_1 = 3, k_2 = 3,\ N = 4\ input\ channels = 3$, then $(3 * 3 * 3 + 1) * 4 = 112$



Number of Parameters

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Chain Rule over all Output Pixels

Need to incorporate all upstream gradients:

$$\left\{ \frac{\partial L}{\partial y(0,0)}, \frac{\partial L}{\partial y(0,1)}, \ldots, \frac{\partial L}{\partial y(H,W)} \right\}$$

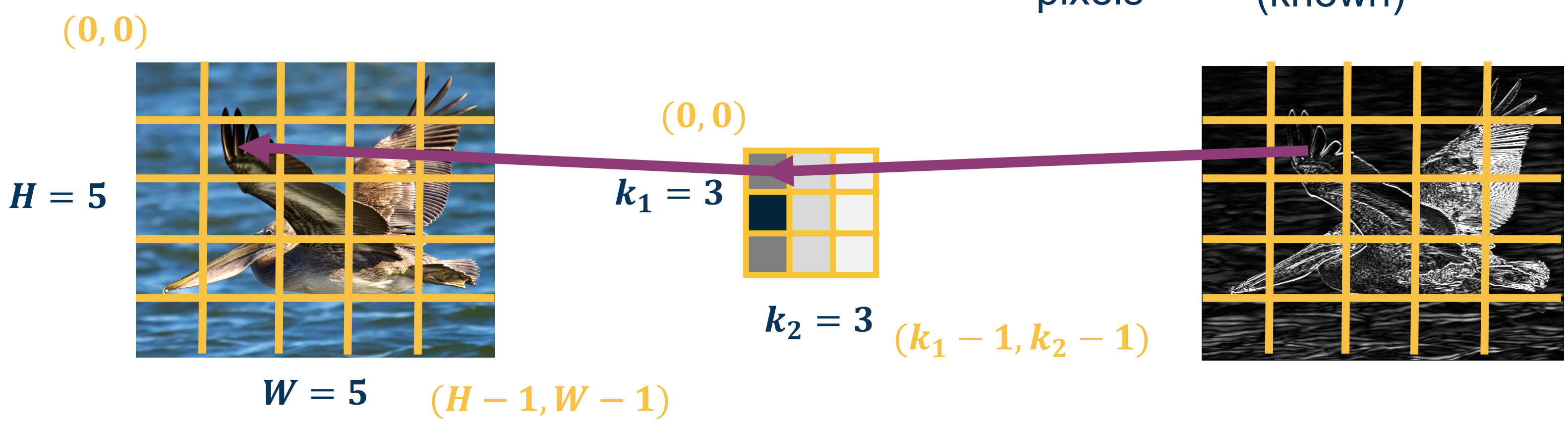
Chain Rule:

$$\frac{\partial L}{\partial k(a,b)} = \sum_{r=0}^{H-1} \sum_{c=0}^{W-1} \frac{\partial L}{\partial y(r,c)} \frac{\partial y(r,c)}{\partial k(a,b)}$$

Sum over all output pixels

Upstream gradient (known)

We will compute



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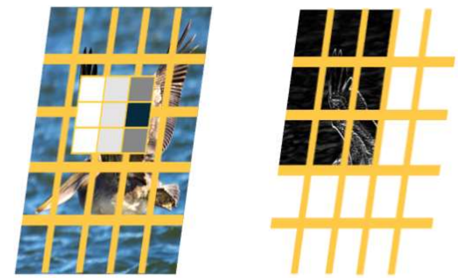
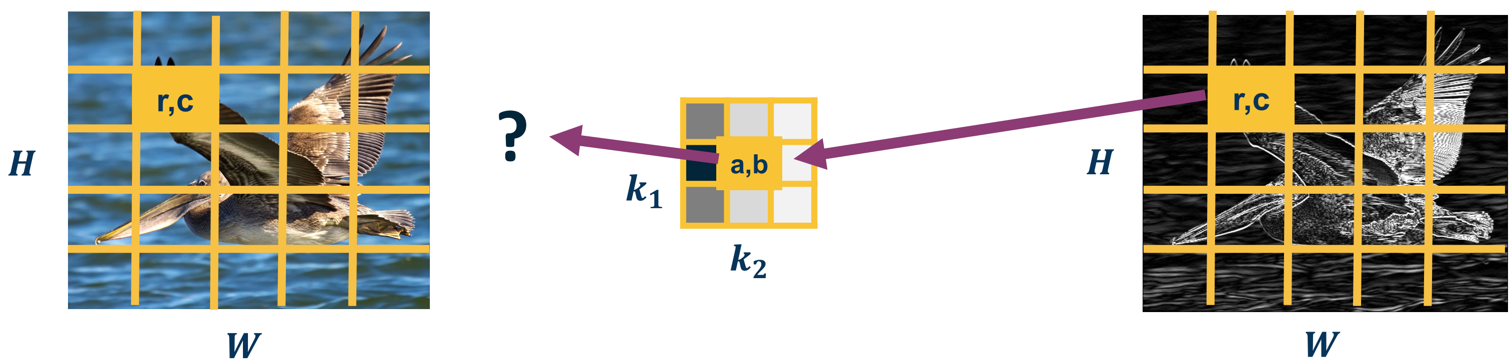
$$\frac{\partial y(r,c)}{\partial k(a,b)} = ?$$



Reasoning:

- Cross-correlation is just "dot product" of kernel and input patch (weighted sum)
- When at pixel $y(r,c)$, kernel is on input *x* such that $k(0,0)$ is multiplied by x(r,c)
- But we want derivative w.r.t. $k(a,b)$
  - $k(0,0) * x(r,c)$, $k(1,1) * x(r+1,c+1)$, $k(2,2) * x(r+2,c+2)$ => in general $k(a,b) * x(r+a,c+b)$
  - Just like before in fully connected layer, partial derivative w.r.t. $k(a,b)$ *only* has this term (other *x* terms go away because not multiplied by $k(a,b)$).



Chain Rule over all Output Pixels

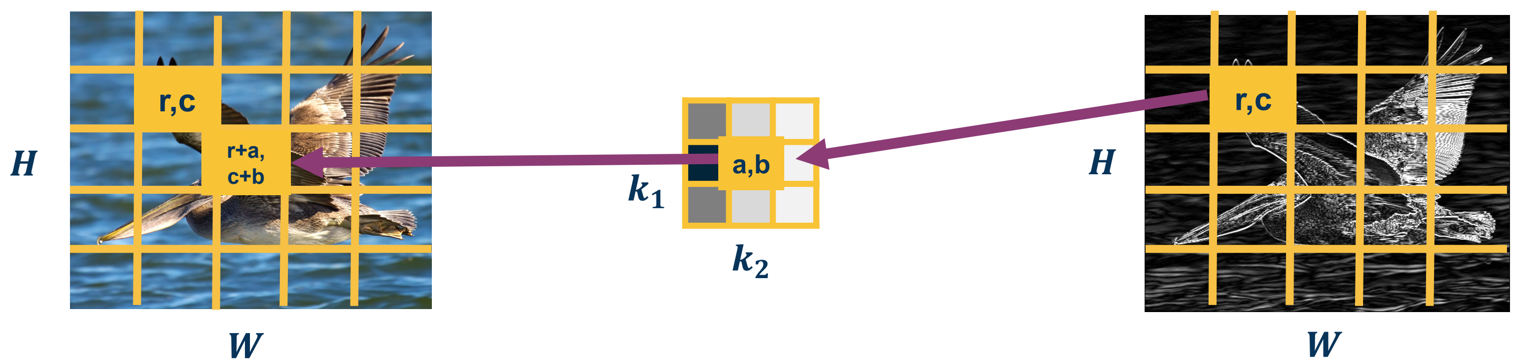
$$\frac{\partial y(r,c)}{\partial k(a,b)} = x(r+a, c+b)$$

$$\frac{\partial L}{\partial k(a,b)} = \sum_{r=0}^{H-1}\sum_{c=0}^{W-1}\frac{\partial L}{\partial y(r,c)}x(r+a, c+b)$$

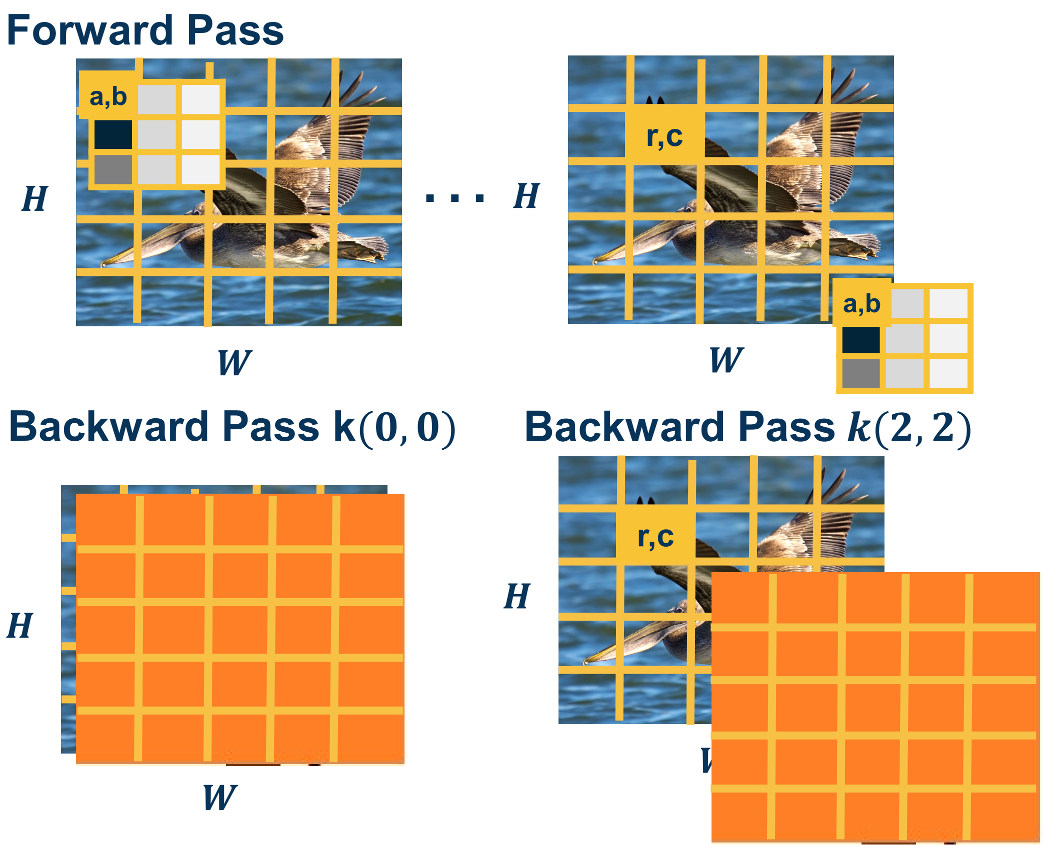
Does this look familiar?

Cross-correlation between upstream gradient and input!

(until $k_1 \times k_2$ output)



Gradients and Cross-Correlation



Does this look familiar?

Cross-correlation between upstream gradient and input!

(until $k_1 \times k_2$ output)

Forward and Backward Duality

$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial y} \frac{\partial y}{\partial x}$$

Gradient for input (to pass to prior layer)

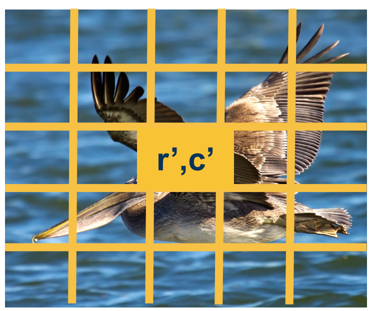
Calculate one pixel at a time $\frac{\partial L}{\partial x(r', c')}$

What does this input pixel affect at the output?

Neighborhood around it (where part of the kernel touches it)

$(0, 0)$

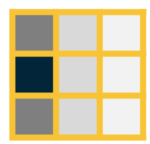
$H = 5$



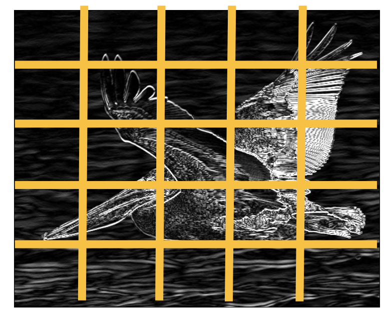
$W = 5$ $(H-1, W-1)$

$(0, 0)$

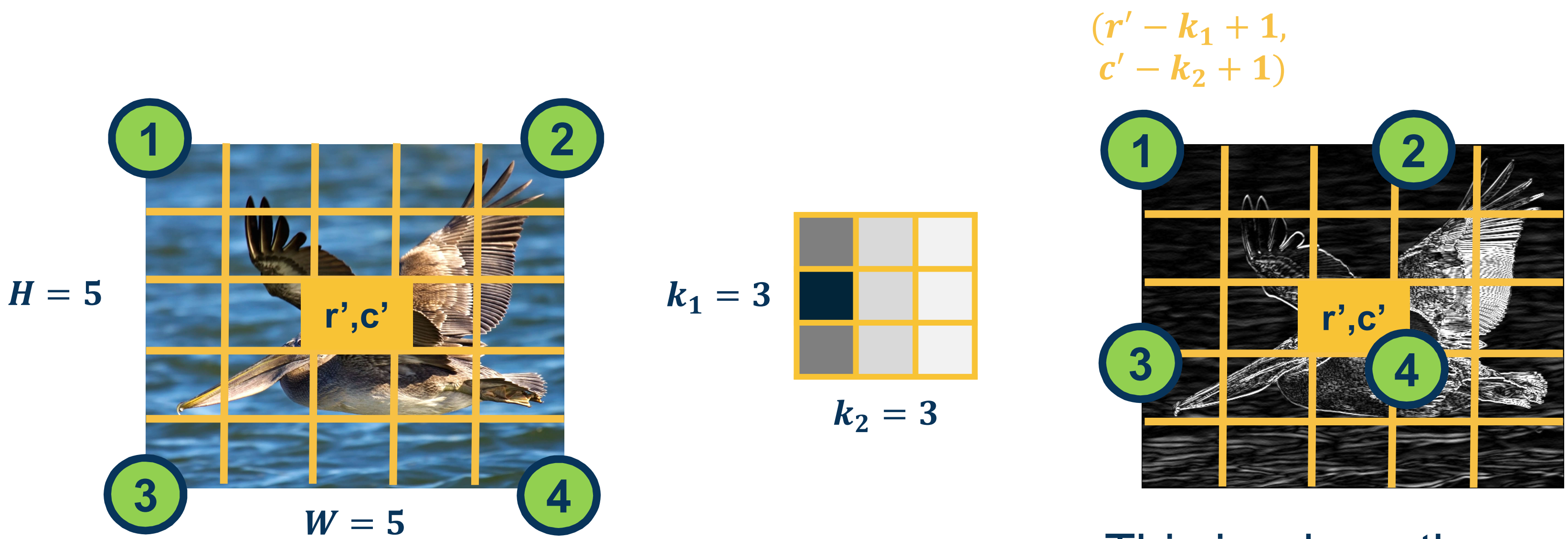
$k_1 = 3$



$k_2 = 3$ $(k_1 - 1, k_2 - 1)$



What an Input Pixel Affects at Output



This is where the corresponding locations are for the **output**

Extents at the Output

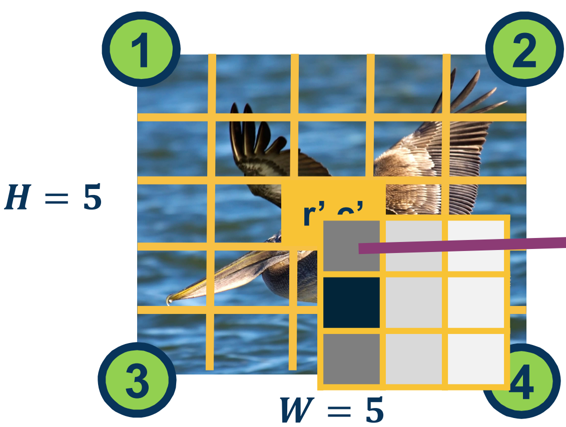
Summing Gradient Contributions

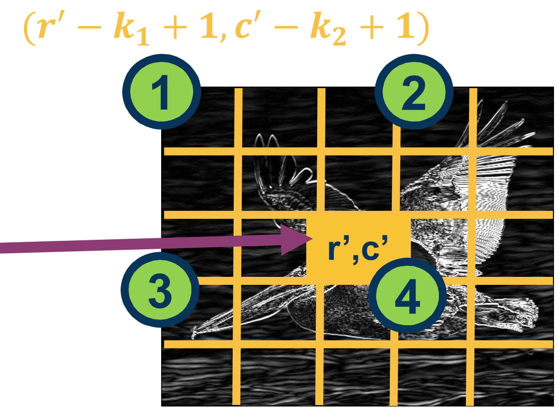
Chain rule for affected pixels (sum gradients):

$$\frac{\partial L}{\partial x(r',c')} = \sum_{Pixels\ p} \frac{\partial L}{\partial y(p)} \frac{\partial y(p)}{\partial x(r',c')}$$

$$\frac{\partial L}{\partial x(r',c')} = \sum_{a=0}^{k_1-1} \sum_{b=0}^{k_2-1} \frac{\partial L}{\partial y(?,?)} \frac{\partial y(?,?)}{\partial x(r',c')}$$

$x(r',c') * k(0,0) \Rightarrow y(r',c')$
$x(r',c') * k(1,1) \Rightarrow ?$





Chain rule for affected pixels (sum gradients):

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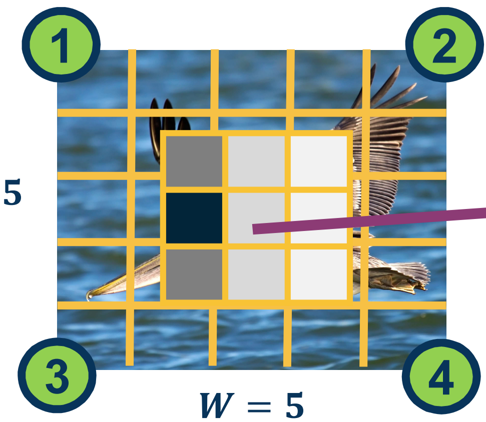
$$\frac{\partial L}{\partial x(r',c')} = \sum_{a=0}^{k_1-1} \sum_{b=0}^{k_2-1} \frac{\partial L}{\partial y(?,?)} \frac{\partial y(?,?)}{\partial x(r',c')}$$

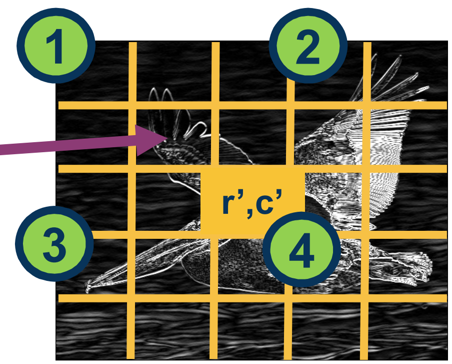
$$x(r',c') * k(0,0) \Rightarrow y(r',c')$$
$$x(r',c') * k(1,1) \Rightarrow y(r'-1,c'-1)$$
$$\ldots$$
$$x(r',c') * k(a,b) \Rightarrow y(r'-a,c'-b)$$

$H = 5$



$W = 5$

$(r'-k_1+1, c'-k_2+1)$



Summing Gradient Contributions

Chain rule for affected pixels (sum gradients):

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$$\frac{\partial L}{\partial x(r',c')} = \sum_{a=0}^{k_1-1} \sum_{b=0}^{k_2-1} \frac{\partial L}{\partial y(r'-a,c'-b)} \frac{\partial y(r'-a,c'-b)}{\partial x(r',c')}$$

Let's derive it analytically this time (as opposed to visually)

Summing Gradient Contributions

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Plugging in to earlier equation:

$$\frac{\partial L}{\partial x(r', c')} = \sum_{a=0}^{k_1-1} \sum_{b=0}^{k_2-1} \frac{\partial L}{\partial y(r'-a, c'-b)} \frac{\partial y(r'-a, c'-b)}{\partial x(r', c')}$$

Does this look familiar?

$$= \sum_{a=0}^{k_1-1} \sum_{b=0}^{k_2-1} \frac{\partial L}{\partial y(r'-a, c'-b)} k(a, b)$$

Convolution between upstream gradient and kernel!

(can implement by flipping kernel and cross- correlation)

Again, all operations can be implemented via matrix multiplications (same as FC layer)!

Backwards is Convolution

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- Convolutions are mathematical descriptions of striding linear operation

- In practice, we implement **cross-correlation neural networks!** (still called convolutional neural networks due to history)
  - Can connect to convolutions via duality (flipping kernel)
  - Convolution formulation has mathematical properties explored in ECE

- Duality for forwards and backwards:
  - **Forward**: Cross-correlation
  - **Backwards w.r.t. K**: Cross-correlation b/w upstream gradient and input
  - **Backwards w.r.t. X**: Convolution b/w upstream gradient and kernel
    - In practice implement via cross-correlation and flipped kernel

- All operations still implemented via **efficient linear algebra** (e.g. matrix-matrix multiplication)

Summary

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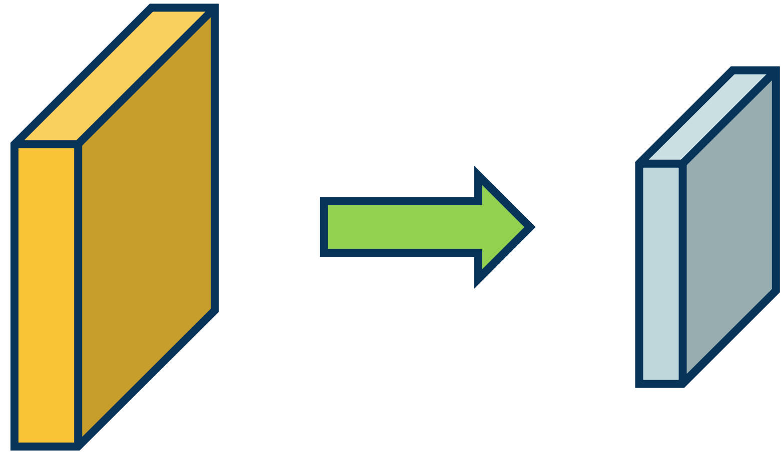
Pooling Layers

Pooling Layers

- **Dimensionality reduction** is an important aspect of machine learning
- Can we make a layer to **explicitly down-sample** image or feature maps?



- **Yes!** We call one class of these operations **pooling** operations

Parameters

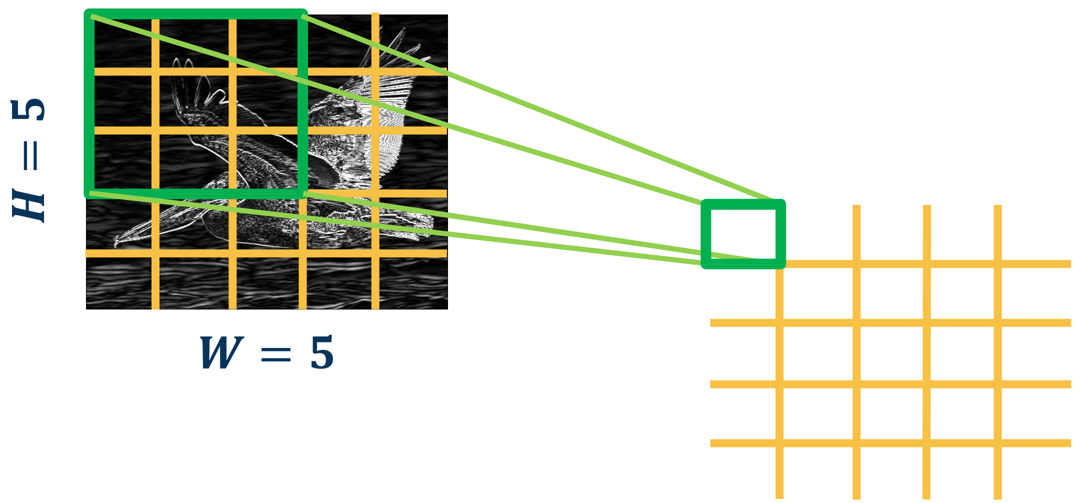
- **kernel_size** – the size of the window to take a max over
- **stride** – the stride of the window. Default value is `kernel_size`
- **padding** – implicit zero padding to be added on both sides

From: https://pytorch.org/docs/stable/generated/torch.nn.MaxPool2d.html#torch.nn.MaxPool2d

Example: Max pooling

- Stride window across image but perform per-patch **max operation**

$$X(0{:}2, 0{:}2) = \begin{bmatrix} 200 & 150 & 150 \\ 100 & 50 & 100 \\ 25 & 25 & 10 \end{bmatrix} \Rightarrow \max(0{:}2,0{:}2) = 200$$



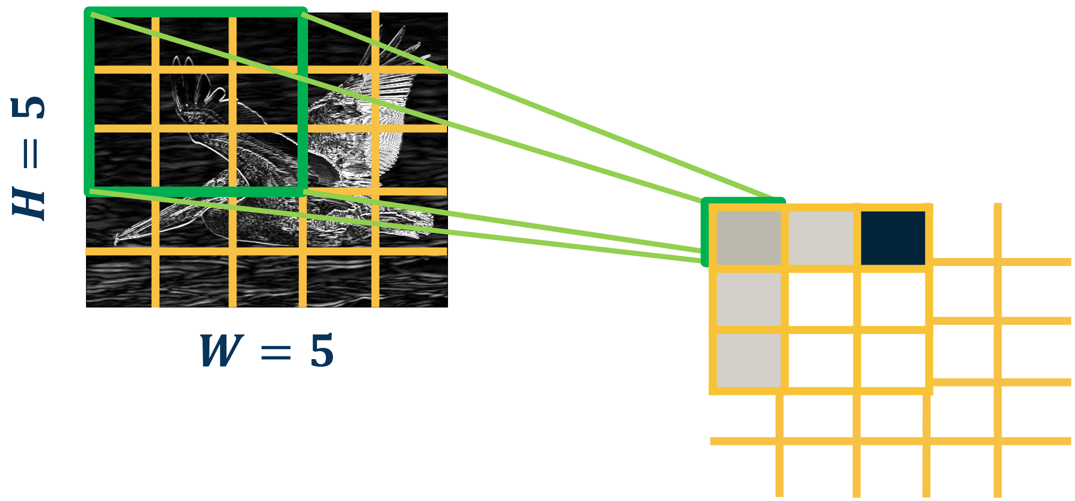
How many learned parameters does this layer have?

None!

Not restricted to max; can use any differentiable function

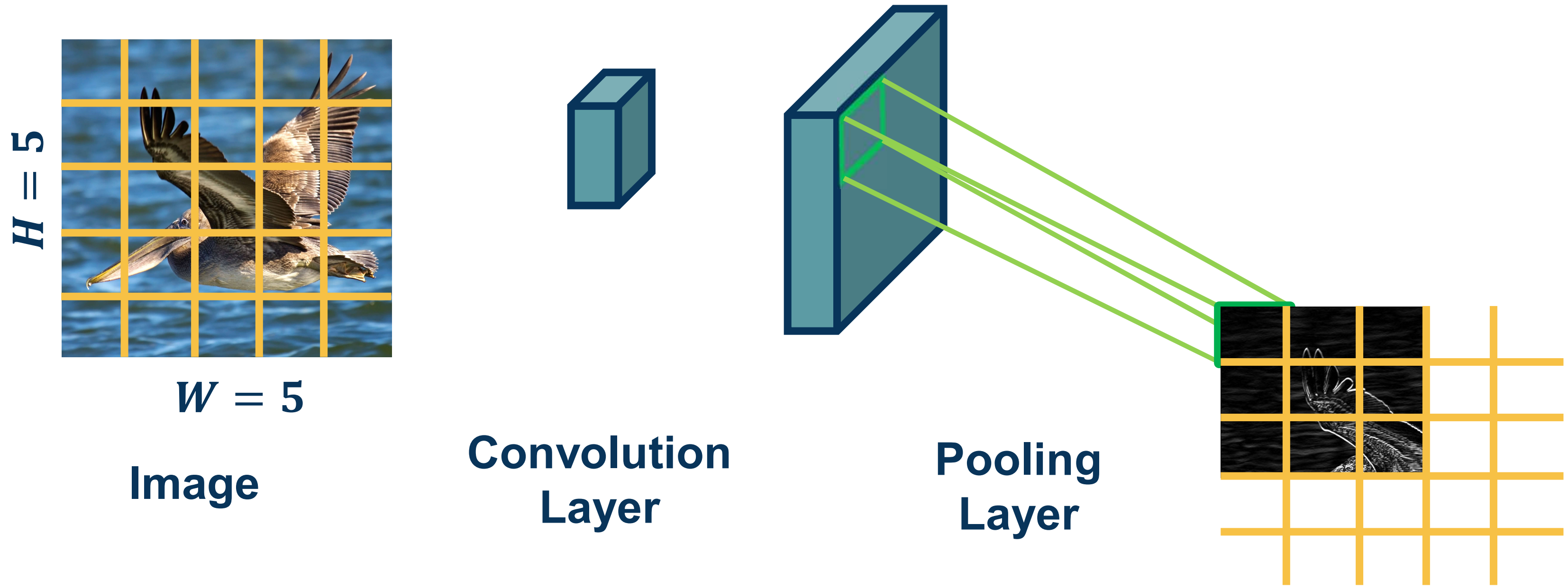
- Not very common in practice

$$X(0:2,0:2) = \begin{bmatrix} 200 & 150 & 150 \\ 100 & 50 & 100 \\ 25 & 25 & 10 \end{bmatrix} \Rightarrow \text{average}(0:2,0:2) = \frac{1}{N}\sum_i \sum_j x(i,j) = 90$$



Max Pooling

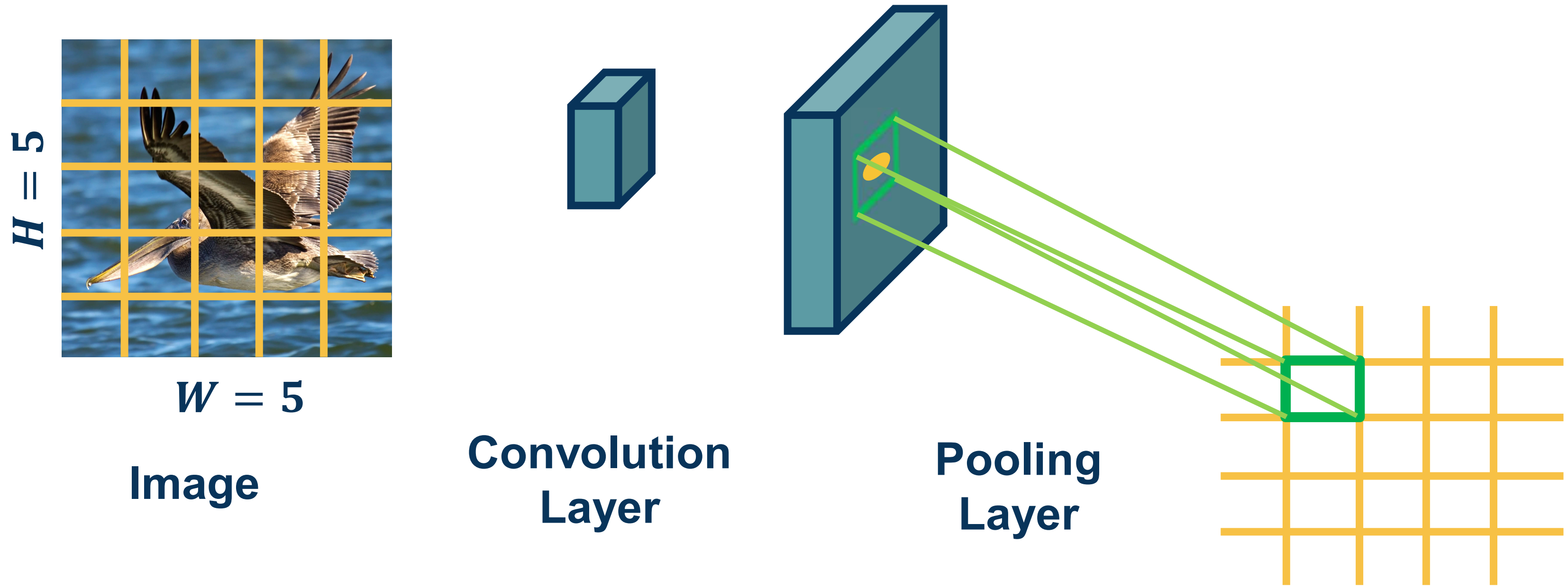
Since the **output** of convolution and pooling layers are **(multi-channel) images**, we can sequence them just as any other layer



Combining Convolution & Pooling Layers

This combination adds some **invariance** to translation of the features

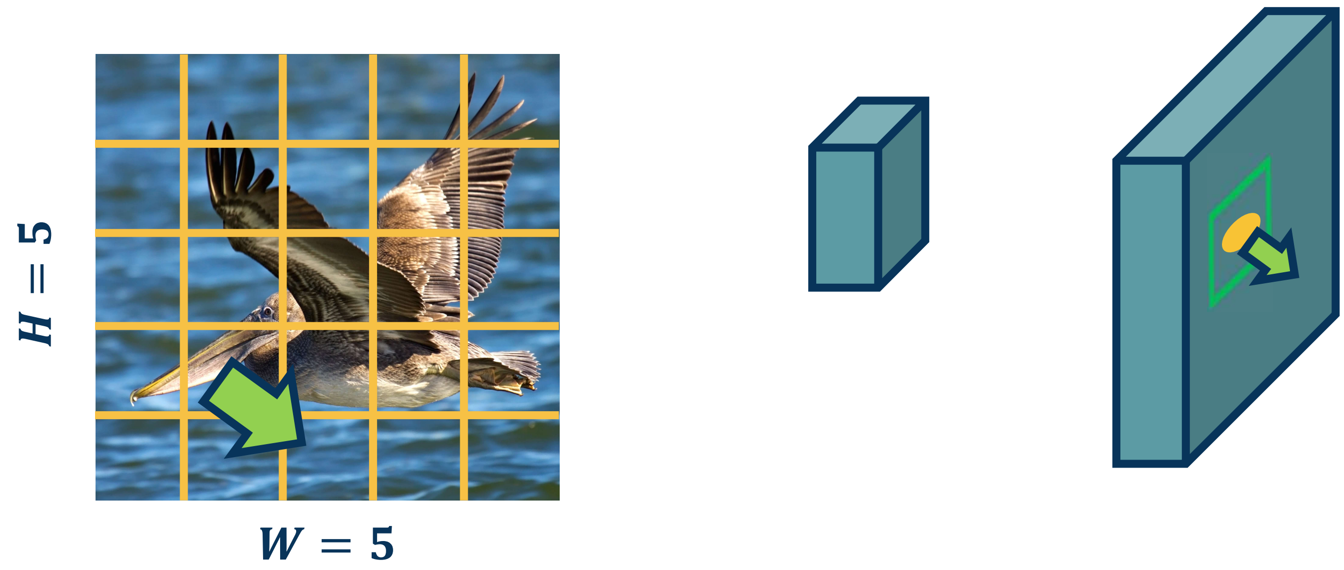
- If feature (such as beak) translated a little bit, output values still **remain the same**



Invariance

Convolution by itself has the property of **equivariance**

- If feature (such as beak) translated a little bit, output values **move by the same translation**



Invariance vs. Equivariance

Simple Convolutional Neural Networks

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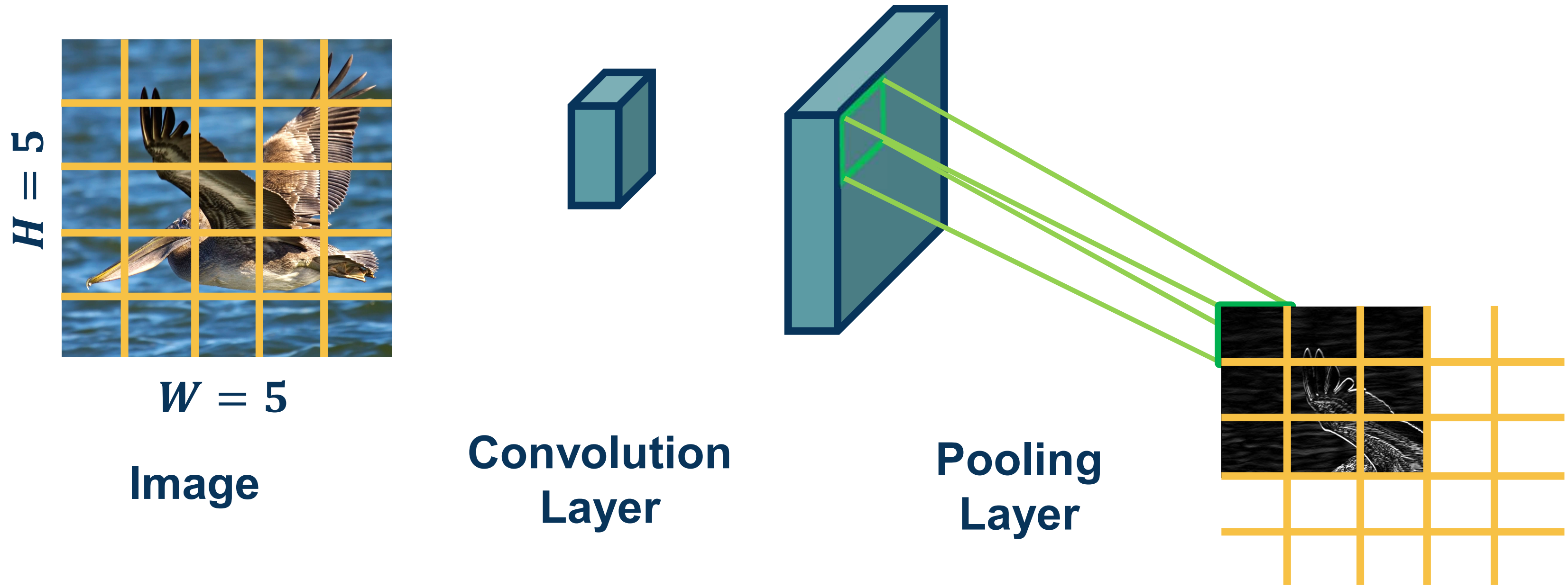
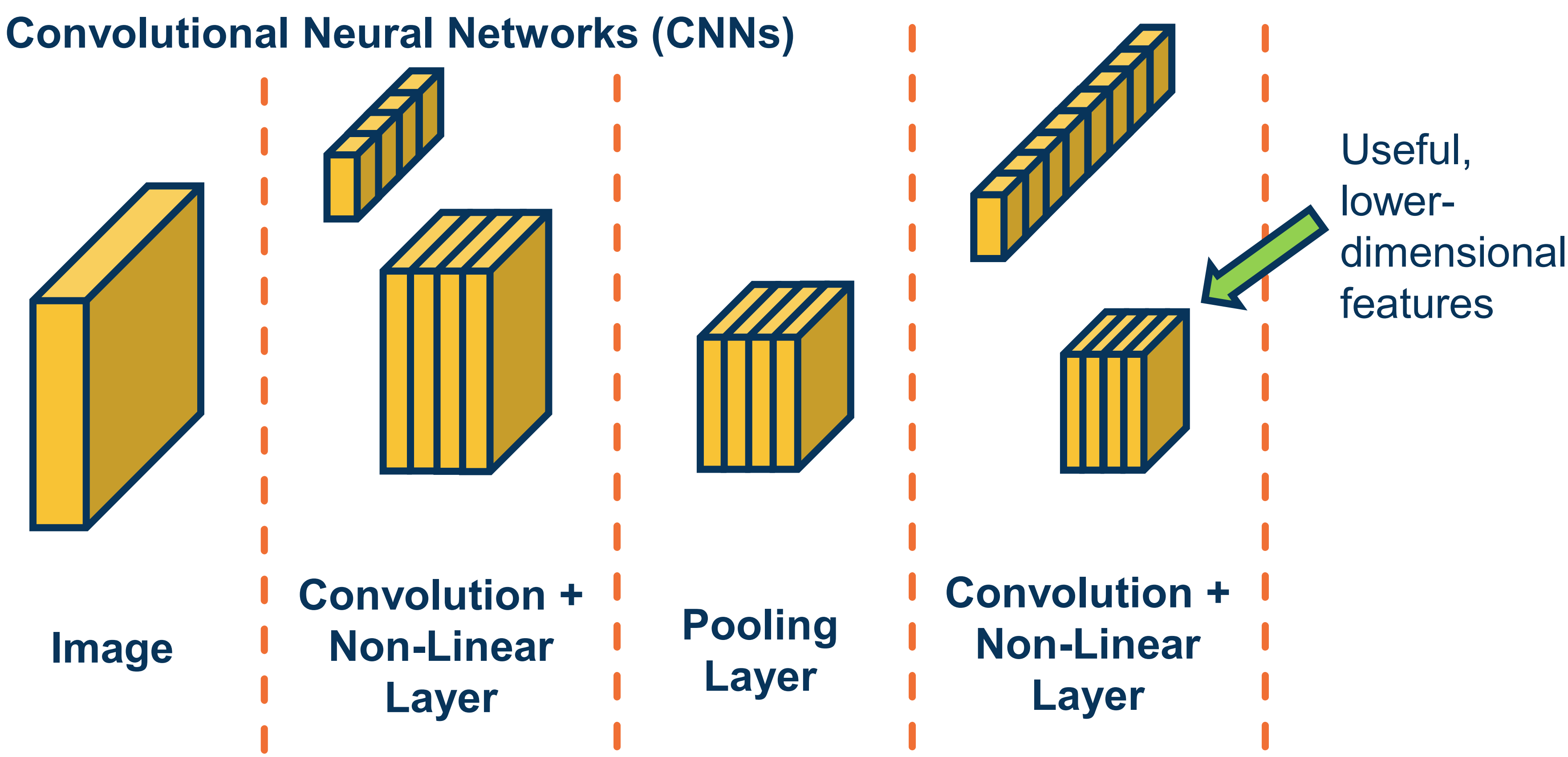
Since the **output** of convolution and pooling layers are **(multi-channel) images**, we can sequence them just as any other layer



Combining Convolution & Pooling Layers



Alternating Convolution and Pooling

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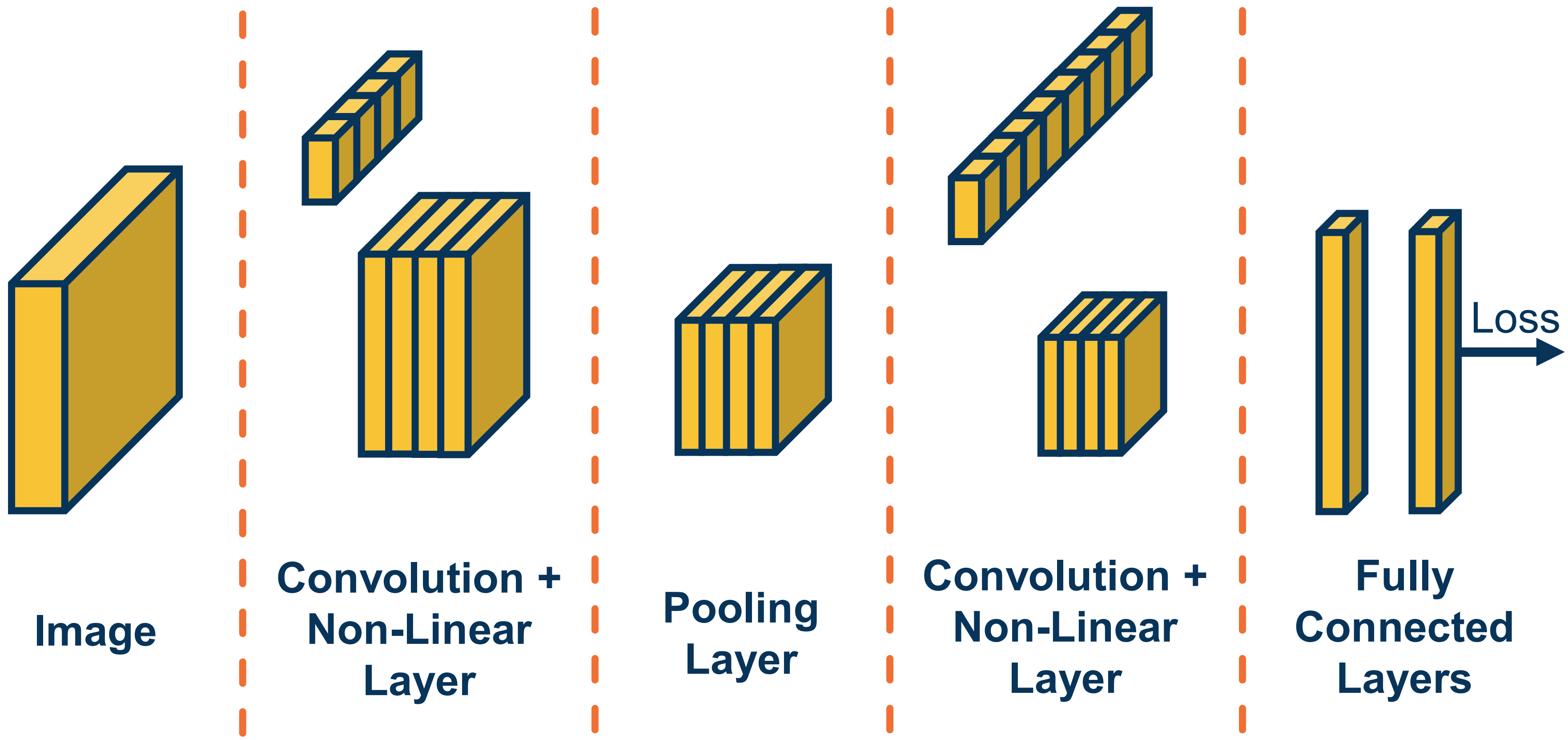
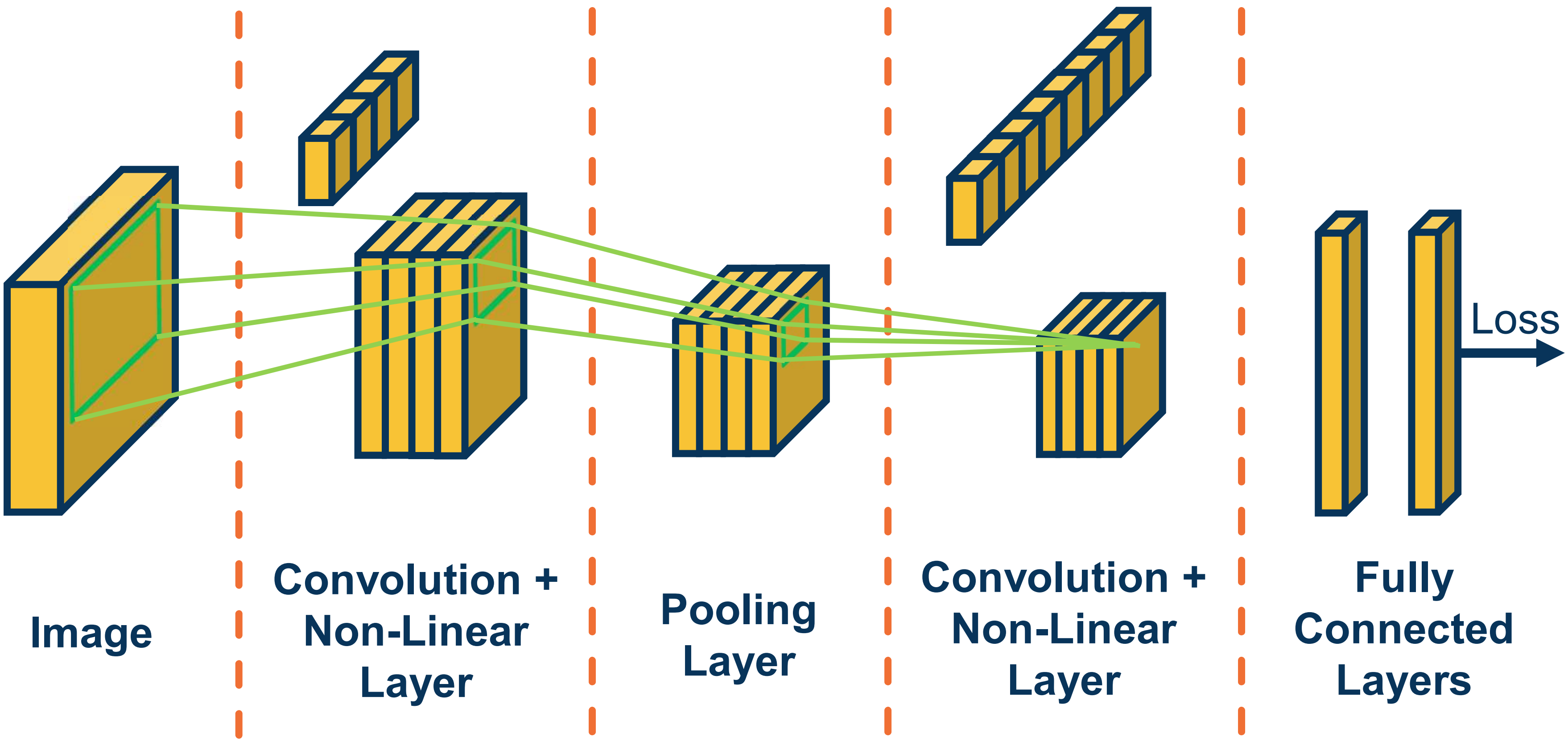
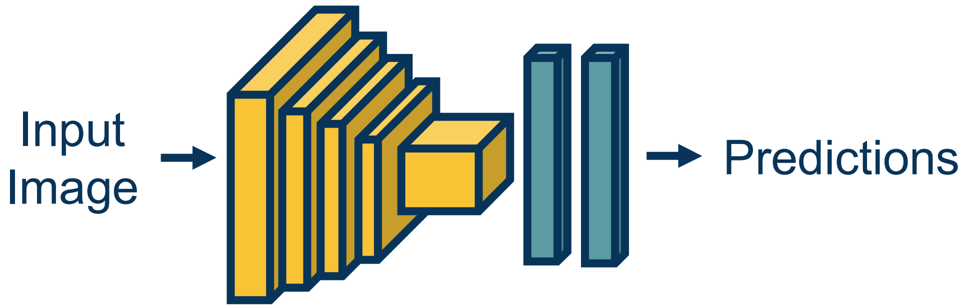


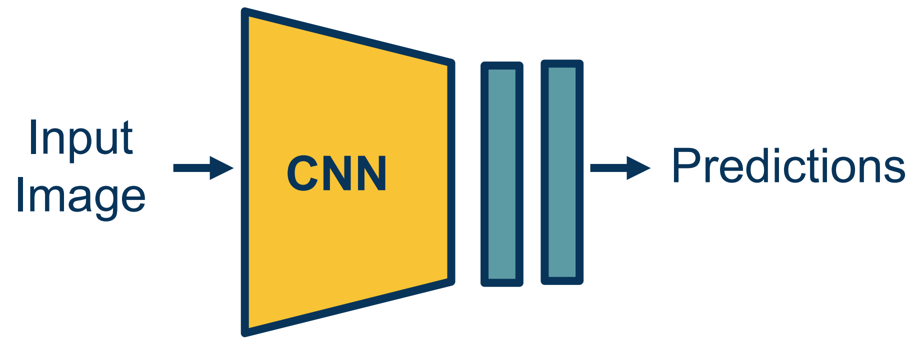
Adding a Fully Connected Layer



Receptive Fields



Convolutional Neural Networks



Typical Depiction of CNNs

These architectures have existed **since 1980s**

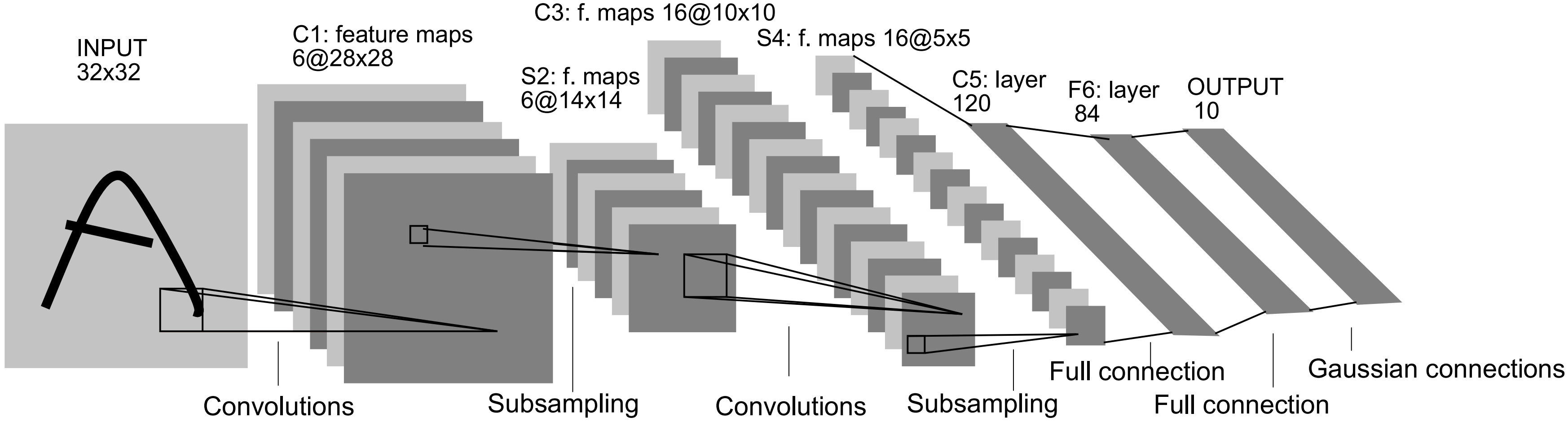


Image Credit: Yann LeCun, Kevin Murphy

LeNet Architecture

Handwriting Recognition

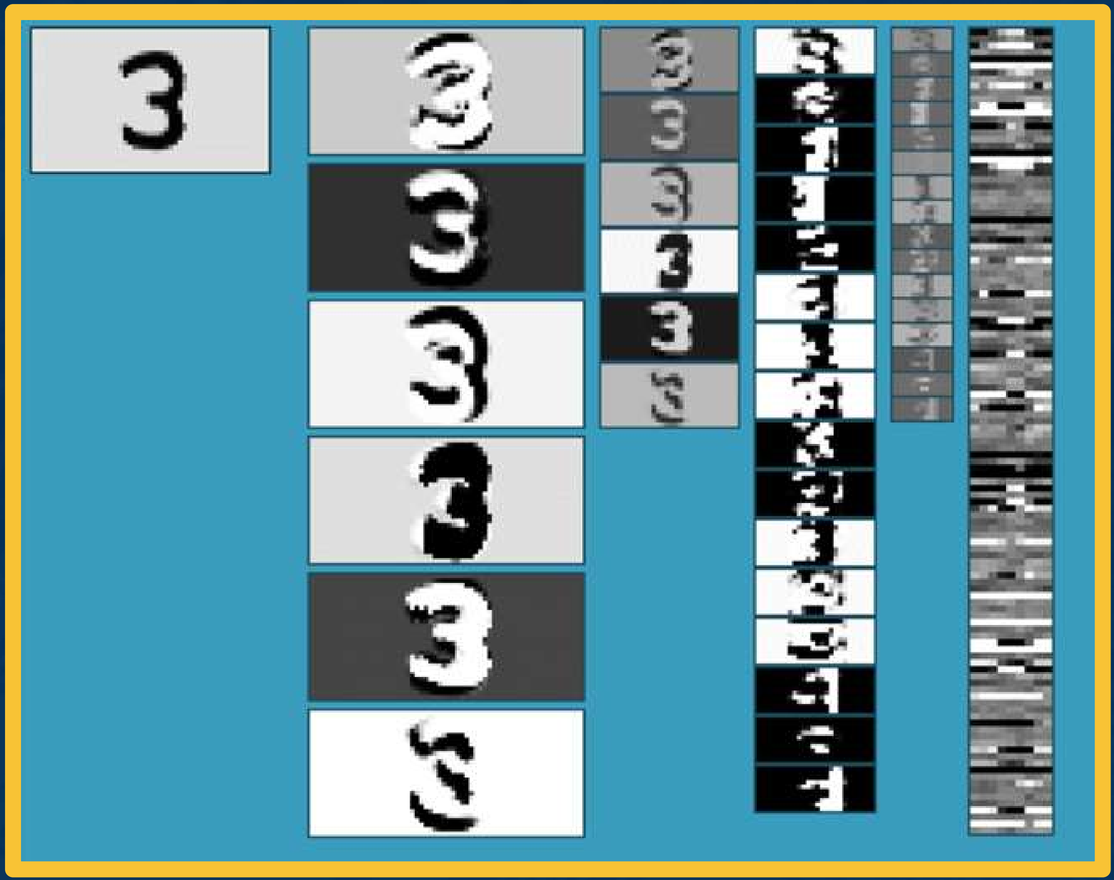


Image Credit: Yann LeCun

Translation Equivariance (Conv Layers) & Invariance (Output)

Image Credit: Yann LeCun

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(Some) Rotation Invariance

Image Credit: Yann LeCun

(Some) Scale Invariance

Image Credit: Yann LeCun

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Advanced Convolutional Networks

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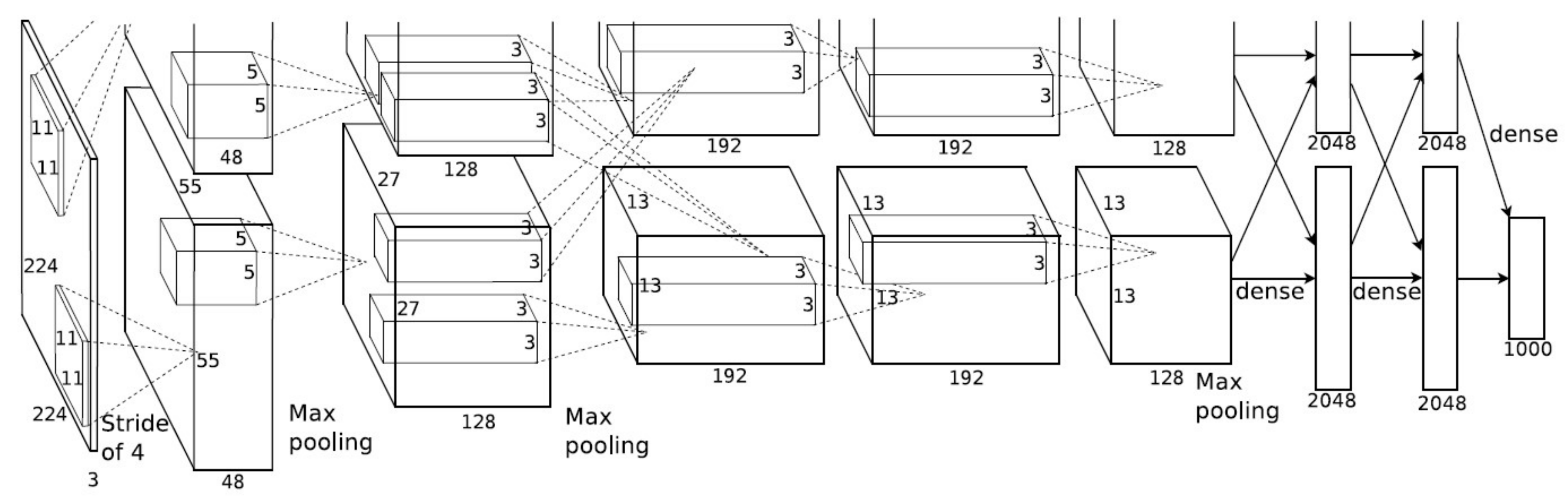
The Importance of Benchmarks

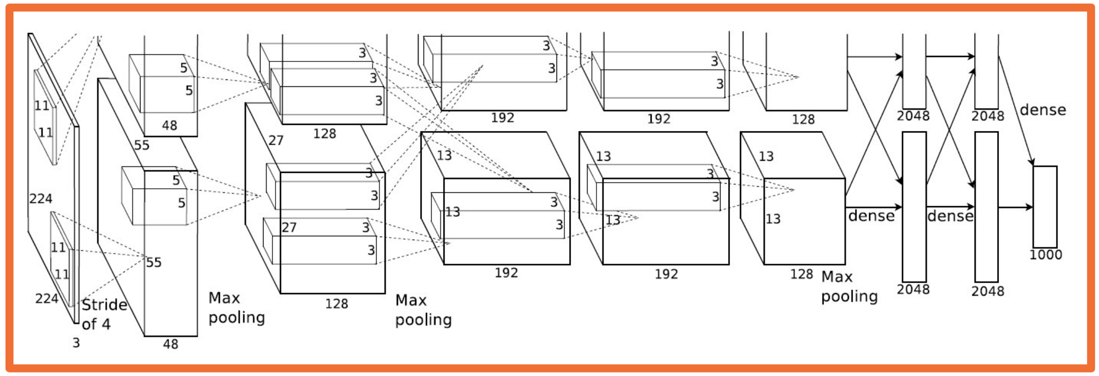
From: https://paperswithcode.com

AlexNet - Architecture



From: Krizhevsky et al., ImageNet Classification with Deep ConvolutionalNeural Networks, 2012.

Full (simplified) AlexNet architecture:
[224x224x3] INPUT
[55x55x96] CONV1: 96 11x11 filters at stride 4, pad 0
[27x27x96] MAX POOL1: 3x3 filters at stride 2
[27x27x96] NORM1: Normalization layer
[27x27x256] CONV2: 256 5x5 filters at stride 1, pad 2
[13x13x256] MAX POOL2: 3x3 filters at stride 2
[13x13x256] NORM2: Normalization layer
[13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1
[13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1
[13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1
[6x6x256] MAX POOL3: 3x3 filters at stride 2
[4096] FC6: 4096 neurons
[4096] FC7: 4096 neurons
[1000] FC8: 1000 neurons (class scores)



Key aspects:

- ReLU instead of sigmoid or tanh
- Specialized normalization layers
- PCA-based data augmentation
- Dropout
- Ensembling

From: Slides by Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

AlexNet – Layers and Key Aspects

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INPUT: [224x224x3] memory: 224*224*3=150K params: 0 (not counting biases)
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*3)*64 = 1,728
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*64)*64 = 36,864
POOL2: [112x112x64] memory: 112*112*64=800K params: 0
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456
POOL2: [56x56x128] memory: 56*56*128=400K params: 0
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
POOL2: [28x28x256] memory: 28*28*256=200K params: 0
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
POOL2: [14x14x512] memory: 14*14*512=100K params: 0
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
POOL2: [7x7x512] memory: 7*7*512=25K params: 0
FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = 102,760,448
FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216
FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4,096,000

ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224 × 224 RGB image)					
conv3-64	conv3-64 **LRN**	conv3-64 **conv3-64**	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 **conv3-128**	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 **conv1-256**	conv3-256 conv3-256 **conv3-256**	conv3-256 conv3-256 conv3-256 **conv3-256**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

Table 2: **Number of parameters** (in millions).

Network	A,A-LRN	B	C	D	E
Number of parameters	133	133	134	138	144

From: Simonyan & Zimmerman, Very Deep Convolutional Networks for Large-Scale Image Recognition
From: Slides by Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

VGG

INPUT: [224x224x3] memory: 224*224*3=150K params: 0 (not counting biases)
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*3)*64 = 1,728
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*64)*64 = 36,864
POOL2: [112x112x64] memory: 112*112*64=800K params: 0
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456
POOL2: [56x56x128] memory: 56*56*128=400K params: 0
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
POOL2: [28x28x256] memory: 28*28*256=200K params: 0
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
POOL2: [14x14x512] memory: 14*14*512=100K params: 0
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
POOL2: [7x7x512] memory: 7*7*512=25K params: 0
FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = 102,760,448
FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216
FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4,096,000

Most memory usage in convolution layers

Most parameters in FC layers

From: Simonyan & Zimmerman, Very Deep Convolutional Networks for Large-Scale Image Recognition
From: Slides by Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Parameters and Memory

VGG – Key Characteristics

Key aspects:

Repeated application of:

- 3x3 conv (stride of 1, padding of 1)
- 2x2 max pooling (stride 2)

Very large number of parameters

ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224 × 224 RGB image)					
conv3-64	conv3-64 **LRN**	conv3-64 **conv3-64**	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 **conv3-128**	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 **conv1-256**	conv3-256 conv3-256 **conv3-256**	conv3-256 conv3-256 conv3-256 **conv3-256**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

Table 2: **Number of parameters** (in millions).

Network	A,A-LRN	B	C	D	E
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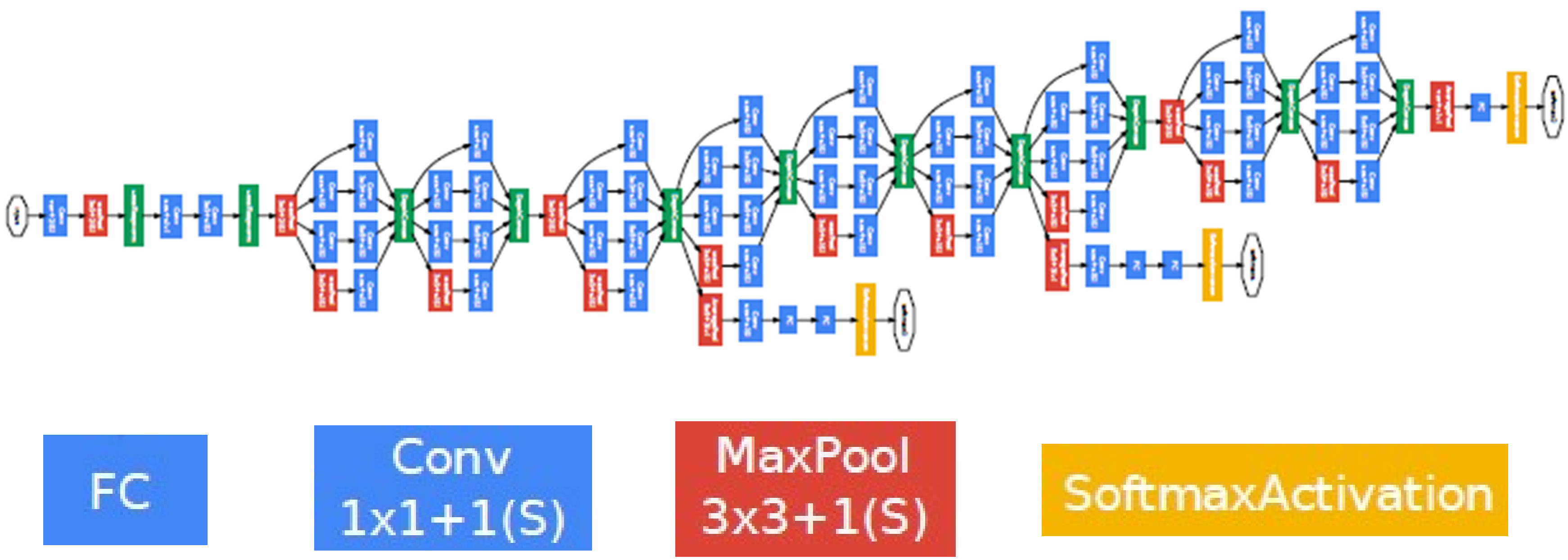
From: Simonyan & Zimmerman, Very Deep Convolutional Networks for Large-Scale Image Recognition
From: Slides by Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

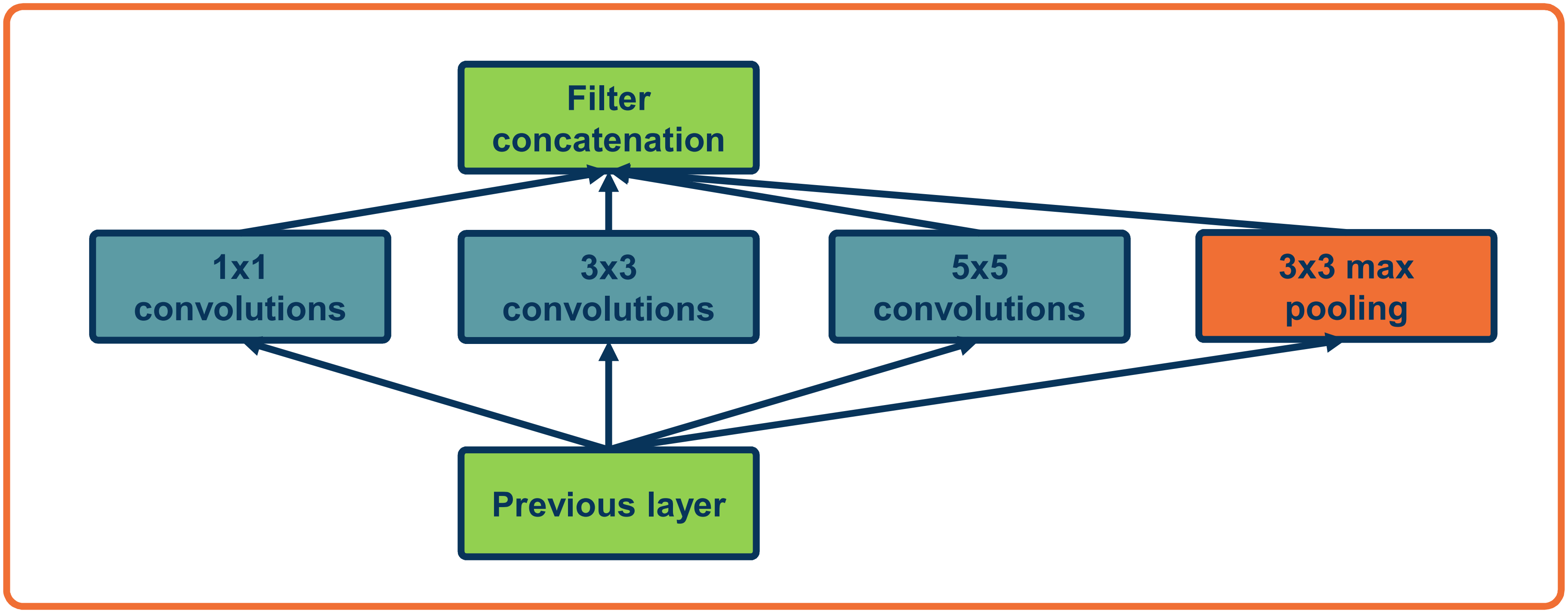
But have become **deeper and more complex**



From: Szegedy et al. Going deeper with convolutions

Inception Architecture

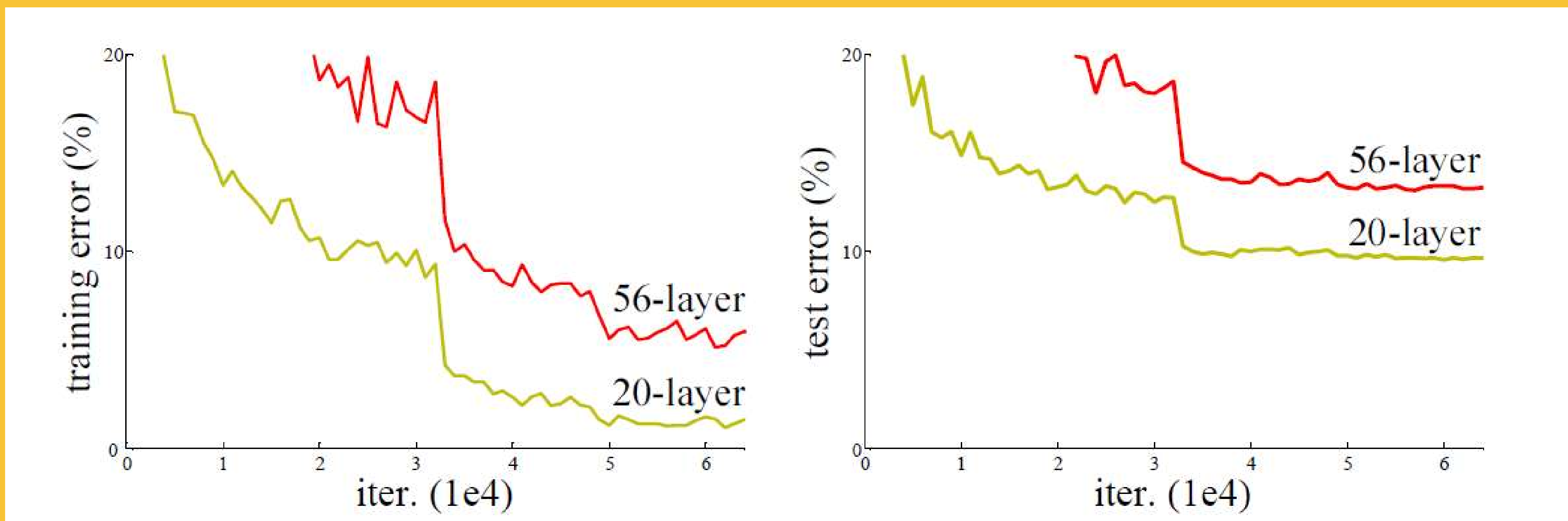
Key idea: Repeated blocks and multi-scale features



From: Szegedy et al. Going deeper with convolutions

Inception Module

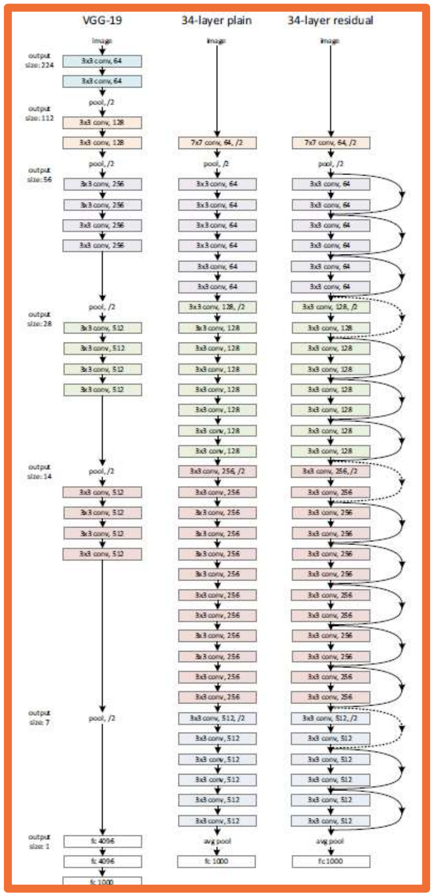
The Challenge of Depth

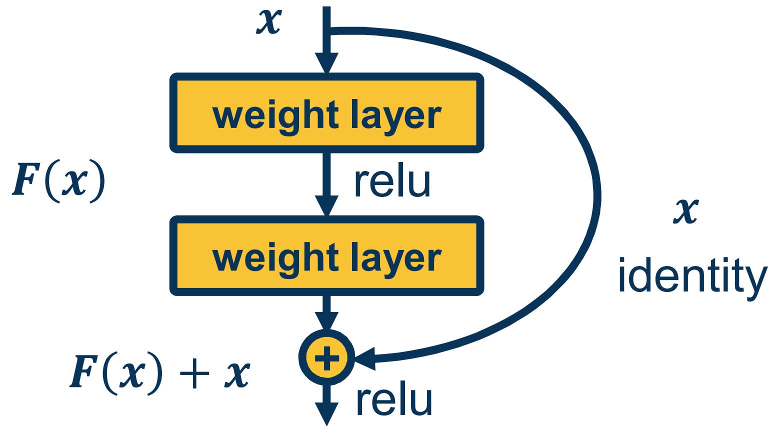


From: He et al., Deep Residual Learning for Image Recognition

Optimizing very deep networks is challenging!

Georgia Tech





Key idea: Allow information from a layer to propagate to any future layer (forward)

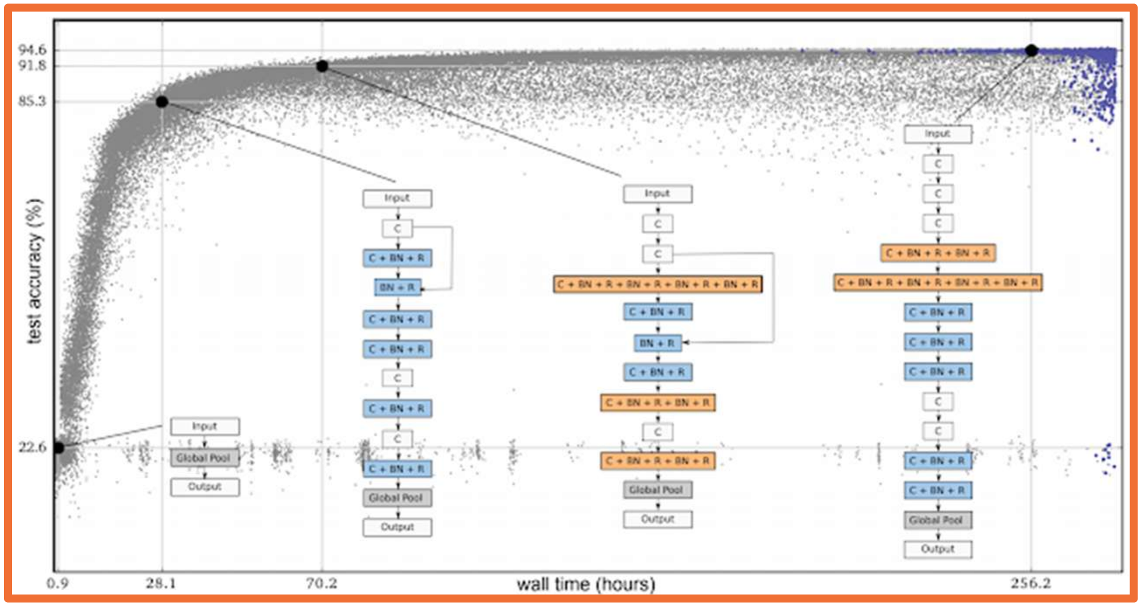
Same is true for gradients!

From: He et al., Deep Residual Learning for Image Recognition

Residual Blocks and Skip Connections

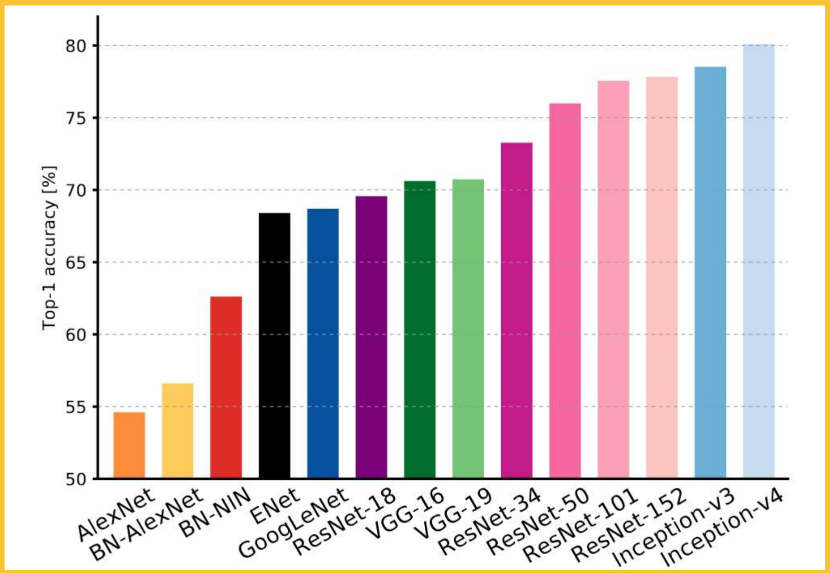
Several ways to *learn* architectures:

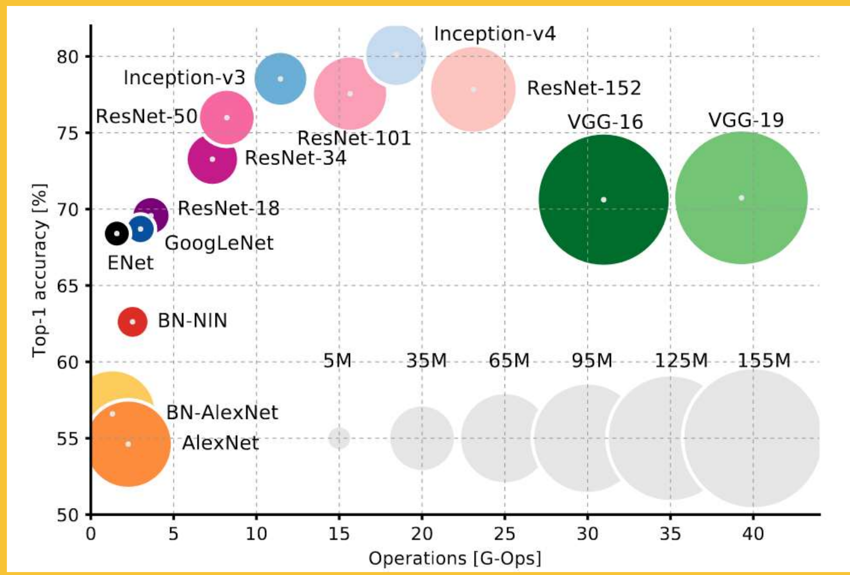
- Evolutionary learning and reinforcement learning
- Prune over-parameterized networks
- Learning of **repeated blocks** typical



From: https://ai.googleblog.com/2018/03/using-evolutionary-automl-to-discover.html

Computational Complexity





From: An Analysis Of Deep Neural Network Models For Practical Applications

Georgia Tech