CS 4803-DL / 7643-A: LECTURE 20
DANFEI XU

Topics:

- Reinforcement Learning Part 1
    - **Markov Decision Processes**
    - **Value Iteration**
    - **(Deep) Q Learning**

Administrative

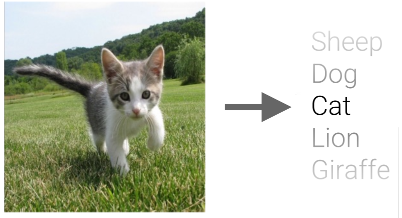
- HW4 is due EOD 11/11. Grace period ends 11/13
- HW3 grades will be released by the end of this week
- Milestone Report grades and feedback will be released by Sunday, 11/13

Reinforcement Learning Introduction

Types of Machine Learning

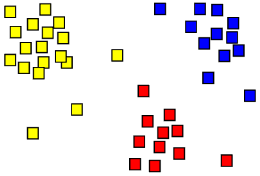
Supervised Learning

- Train Input: $\{X, Y\}$
- Learning output: $f : X \rightarrow Y,\ P(y|x)$
- e.g. classification



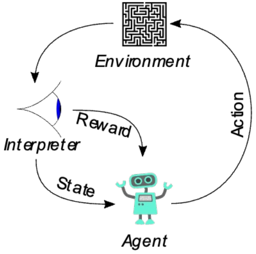
Unsupervised Learning

- Input: $\{X\}$
- Learning output: $P(x)$
- Example: Clustering, density estimation, generative modeling



Reinforcement Learning

- Evaluative feedback in the form of **reward**
- No supervision on the right action



RL: Sequential decision making in an environment with evaluative feedback.

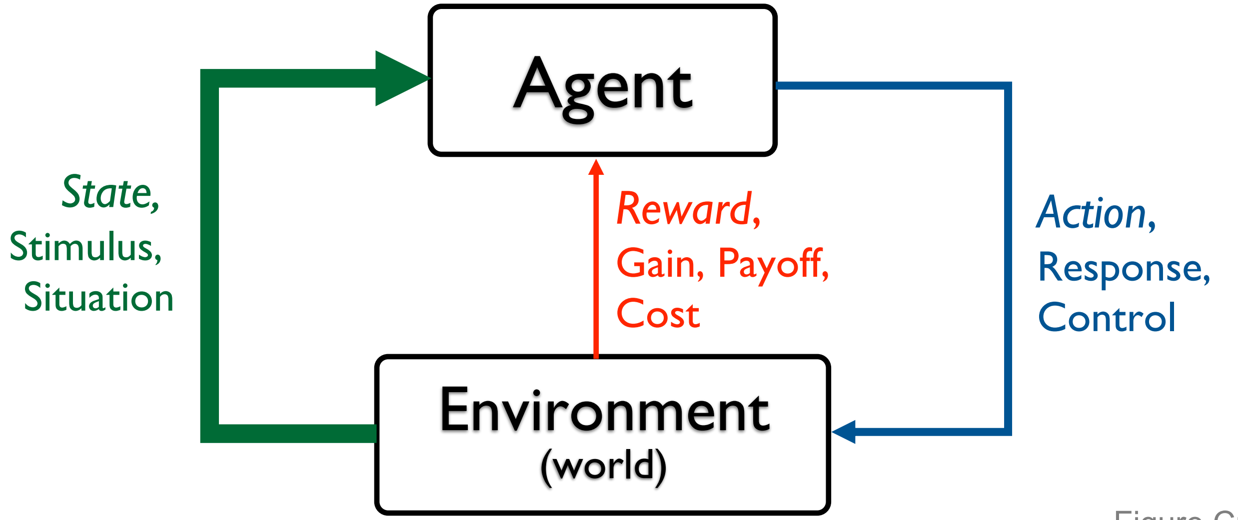
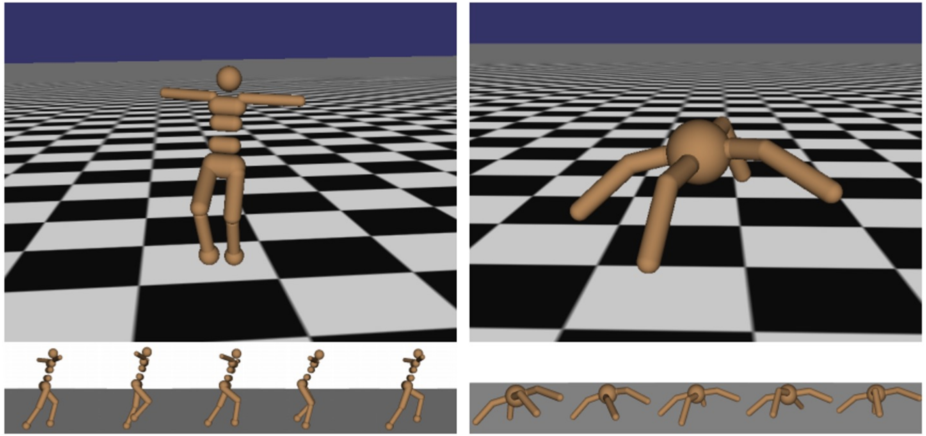


Figure Credit: Rich Sutton

- **Environment** may be unknown, non-linear, stochastic and complex.
- **Agent** learns a **policy** to map states of the environments to actions.
  - Seeking to maximize cumulative reward in the long run.

Example: Robot Locomotion



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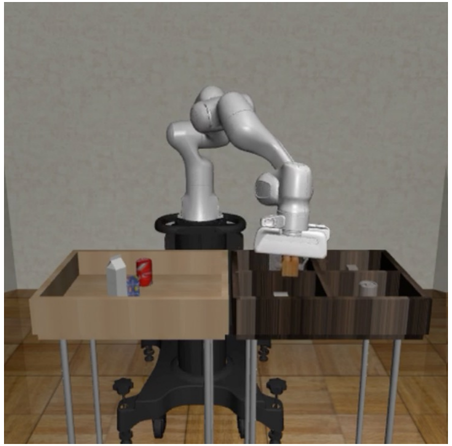
- **Objective**: Make the robot move forward without falling
- **State**: Angle and position of the joints
- **Action**: Torques applied on joints
- **Reward**: +1 at each time step upright and moving forward

Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Example: Robot Manipulation



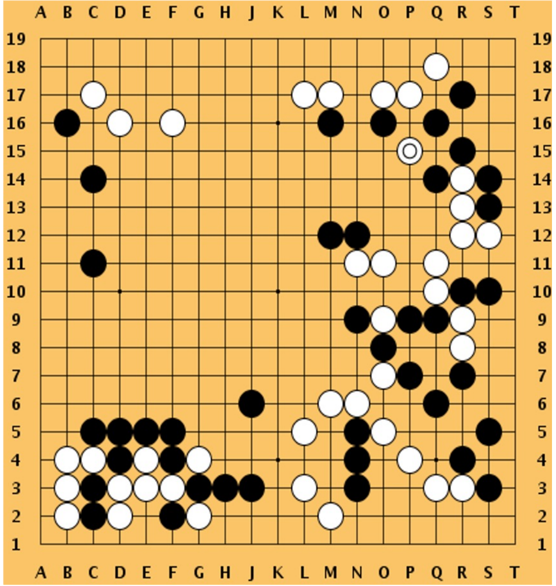
- **Objective**: Pick up object and place to sorting bin
- **State**: Pose of the object and the bin, joint state and velocity of robots
- **Action**: End effector motion
- **Reward**: inverse distance between the object and the bin

Example: Atari Games

- **Objective**: Complete the game with the highest score
- **State**: Raw pixel inputs of the game state
- **Action**: Game controls e.g. Left, Right, Up, Down
- **Reward**: Score increase/decrease at each time step

Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Example: Go



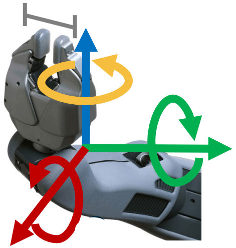
- **Objective**: Defeat opponent
- **State**: Board pieces
- **Action**: Where to put next piece down
- **Reward**: +1 if win at the end of game, 0 otherwise

Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Deep Learning for Decision Making

state
input

Deep Neural
Nets



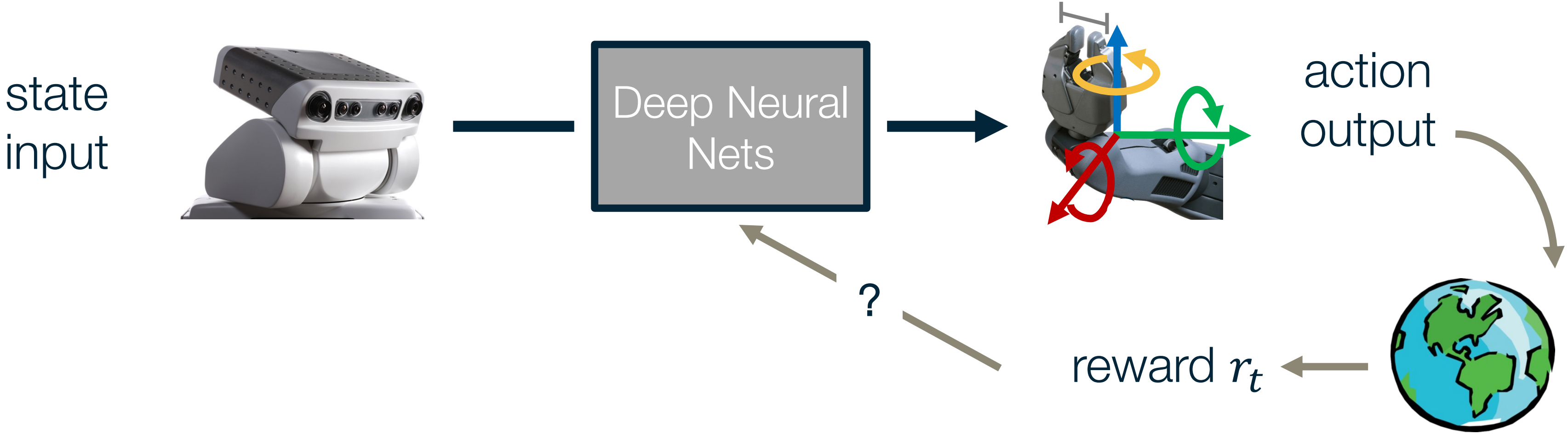
action
output

Deep Learning for Decision Making

Deep Learning for Decision Making

Problem: we don't know the correct action label to supervise the output!

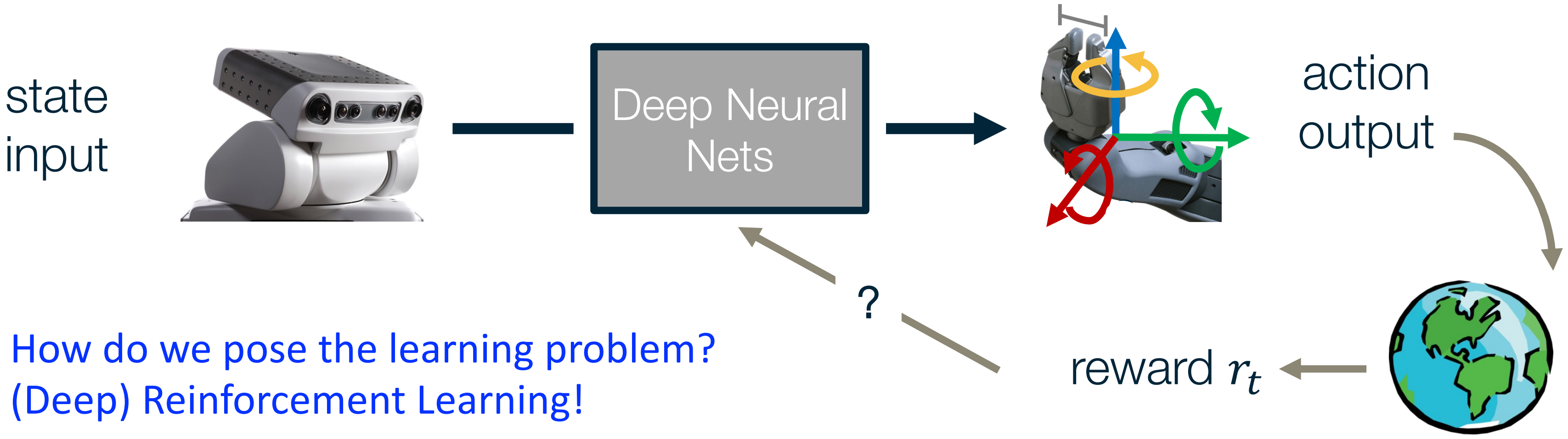
Deep Learning for Decision Making



Problem: we don't know the correct action label to supervise the output!

All we know is the step-wise task reward

Deep Learning for Decision Making



How do we pose the learning problem?
(Deep) Reinforcement Learning!

Problem: we don’t know the correct action label to supervise the output!

All we know is the step-wise task reward

Markov Decision Processes

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- **MDPs**: Theoretical framework underlying RL

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- An MDP is defined as a tuple $(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathbb{T}, \gamma)$

  $\mathcal{S}$: Set of possible states

  $\mathcal{A}$: Set of possible actions

  $\mathcal{R}(s, a, s')$: Distribution of reward

  $\mathbb{T}(s, a, s')$: Transition probability distribution, also written as p(s'|s,a)

  γ : Discount factor

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- **Experience**: $\ldots s_t, a_t, r_{t+1}, s_{t+1}, a_{t+1}, r_{t+2}, s_{t+2}, \ldots$
- **Markov property**: Current state completely characterizes state of the environment
- **Assumption**: Most recent observation is a sufficient statistic of history

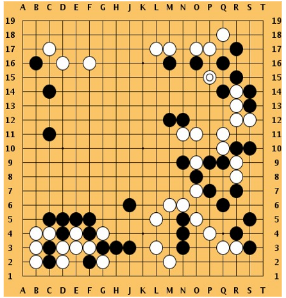
$$p\left(S_{t+1} = s' | S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, \ldots S_0 = s_0\right) = p\left(S_{t+1} = s' | S_t = s_t, A_t = a_t\right)$$

Markov Decision Processes (MDPs)

MDP Variations

Fully observed MDP

- Agent receives the true state s_t at time t
- Example: Chess, Go



Partially observed MDP

- Agent perceives its own partial observation o_t of the state s_t at time t, using past states e.g. with an RNN
- Example: Poker, First-person games (e.g. Doom)

Source: https://github.com/mwydmuch/ViZDoom

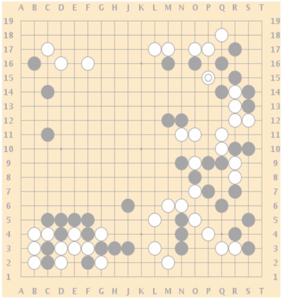
Fully observed MDP

- Agent receives the true state s_t at time t
- Example: Chess, Go

Partially observed MDP

- Agent perceives its own partial observation o_t of the state s_t at time t, using past

We will assume **fully observed MDPs** for this lecture



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- In **Reinforcement Learning**, we assume an underlying **MDP** with unknown:
  - Transition probability distribution $\mathbb{T}$
  - Reward distribution $\mathcal{R}$

MDP
$(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathbb{T}, \gamma)$

MDPs in the context of RL

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- In **Reinforcement Learning**, we assume an underlying **MDP** with unknown:
  - Transition probability distribution $\mathbb{T}$
  - Reward distribution $\mathcal{R}$

MDP
$(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathbb{T}, \gamma)$

Put simply: without learning, the agent doesn't know how their actions will change the environment and what reward they will receive.

Reinforcement learning is to learn the environment transition and (future) reward by actively interacting with the environment and learning from the experience.

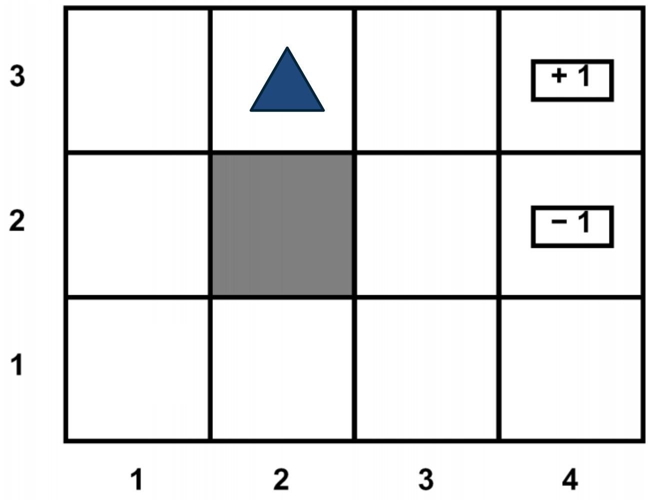
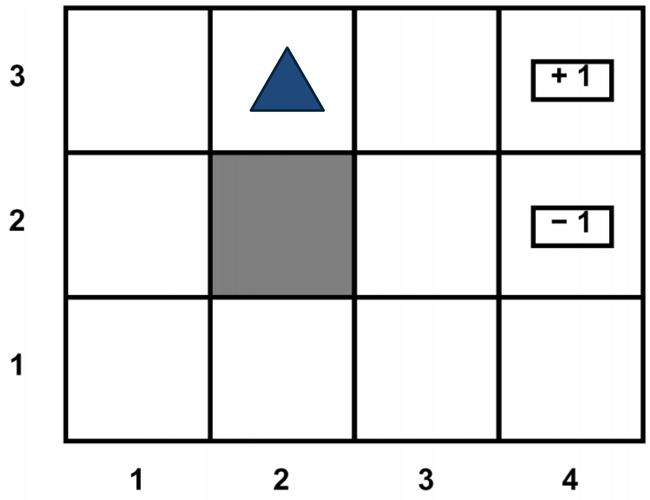


Figure credits: Pieter Abbeel

A Grid World MDP

- Agent lives in a 2D grid environment



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A Grid World MDP

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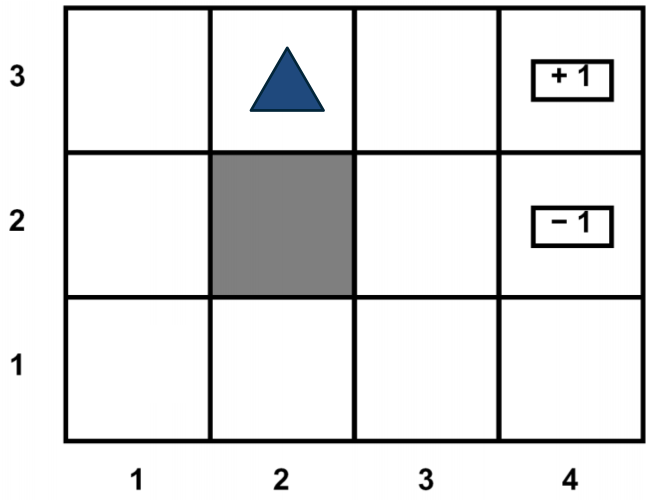
- Agent lives in a 2D grid environment

- State: Agent's 2D coordinates
- Actions: N, E, S, W
- Rewards: +1/-1 at absorbing states



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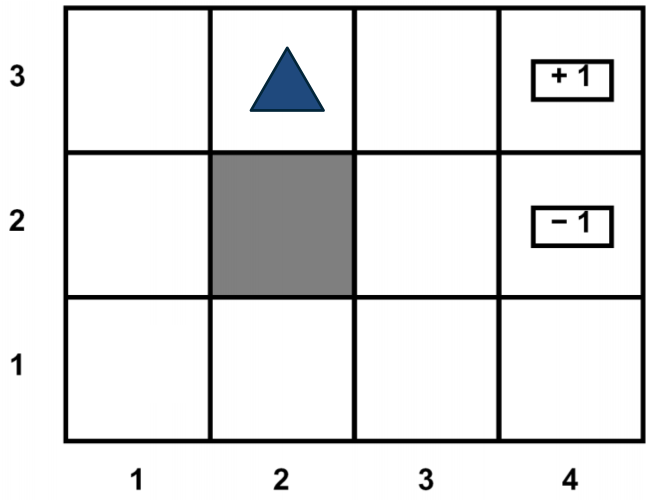
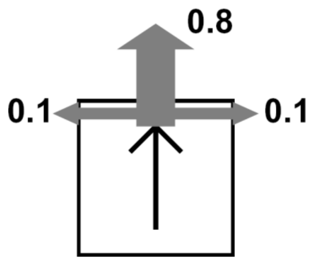
- Agent lives in a 2D grid environment

- State: Agent's 2D coordinates
- Actions: N, E, S, W
- Rewards: +1/-1 at absorbing states

- Walls block agent's path
- Actions to not always go as planned
    - 20% chance that agent drifts one cell left or right of direction of motion (except when blocked by wall).





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A Grid World MDP

- Solving MDPs by finding the **best/optimal policy**

Solving MDPs: Optimal policy

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- Solving MDPs by finding the **best/optimal policy**

- Formally, a **policy** is a mapping from states to actions

e.g.

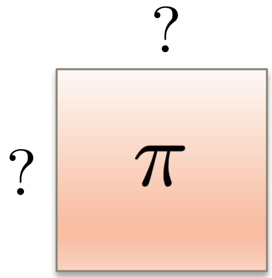
State	Action
A →	2
B →	1

Solving MDPs: Optimal policy

$n = |\mathcal{S}|$

$m = |\mathcal{A}|$

- Solving MDPs by finding the **best/optimal policy**
- Formally, a **policy** is a mapping from states to actions
  - Deterministic $\pi(s) = a$



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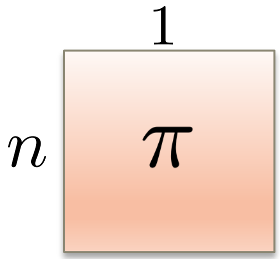

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- Solving MDPs by finding the **best/optimal policy**
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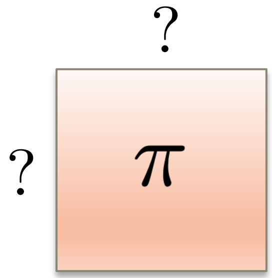

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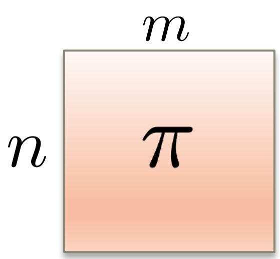

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- What is a good policy?
  - Maximize **current reward**? Sum of all **future rewards**?

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- What is a good policy?
  - Maximize **current reward**? Sum of all **future rewards**?
  - **Discounted sum of future rewards**!
  - How much to value future rewards
    - Discount factor: γ
    - Typically 0.9 - 0.99

- Formally, the **optimal policy** is defined as:

$$\pi^* = \arg\max_{\pi} \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | \pi\right]$$

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$$\boldsymbol{s}_0 \sim p(\boldsymbol{s}_0), a_t \sim \pi(\cdot|\boldsymbol{s}_t), \boldsymbol{s}_{t+1} \sim p(\cdot|\boldsymbol{s}_t, a_t)$$

Expectation over initial state, actions from policy,
next states from transition distribution

- A **value function** predicts the sum of discounted future reward

Value Function

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- A **value function** predicts the sum of discounted future reward

- **State** value function / **V**-function / $V : \mathcal{S} \rightarrow \mathbb{R}$
  - How good is this state?
  - Am I likely to win/lose the game from this state (reward-to-go)?

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- **State-Action** value function / **Q**-function / $Q : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$
  - How good is this state-action pair?
  - In this state, what is the impact of this action on my future?

Value Function

- For a policy that produces a trajectory sample $(s_0, a_0, s_1, a_1, s_2 \cdots)$

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- The **V-function** of the policy at state s, is the expected cumulative reward from state s:

$$V^\pi(s) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | s_0 = s, \pi\right]$$

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↑

?

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Value Function

- For a policy that produces a trajectory sample $(s_0, a_0, s_1, a_1, s_2 \cdots)$

- The **Q-function** of the policy at state **s** and action **a**, is the expected cumulative reward upon taking action **a** in state **s** (and following policy thereafter):

Action-Value Function

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Action-Value Function

- The V and Q functions corresponding to the optimal policy $\pi^\star$

$$V^*(s) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, \pi^*\right]$$

$$Q^*(s,a) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi^*\right]$$

$$\boxed{V^*(s) = \max_a Q^*(s,a)}$$

Optimal V & Q functions

Recursive Bellman expansion (from definition of Q)

(Expected) return from t = 0

$$Q^*(s,a) = \mathop{\mathbb{E}}_{\substack{a_t \sim \pi^*(\cdot|s_t) \\ s_{t+1} \sim p(\cdot|s_t,a_t)}} \left[\sum_{t \geq 0} \gamma^t r(s_t, a_t) \mid s_0 = s, a_0 = a \right]$$

Recursive Bellman expansion (from definition of Q)

(Expected) return from t = 0

$$Q^*(s,a) = \mathbb{E}_{\substack{a_t \sim \pi^*(\cdot|s_t) \\ s_{t+1} \sim p(\cdot|s_t,a_t)}} \left[\sum_{t \geq 0} \gamma^t r(s_t, a_t) \mid s_0 = s, a_0 = a \right]$$

$$= \gamma^0 r(s,a) + \mathbb{E}_{s' \sim p(\cdot|s,a)} \left[\gamma \mathbb{E}_{\substack{a_t \sim \pi^*(\cdot|s_t) \\ s_{t+1} \sim p(\cdot|s_t,a_t)}} \left[\sum_{t \geq 1} \gamma^{t-1} r(s_t, a_t) \mid s_1 = s' \right] \right]$$

$$= r(s,a) + \gamma \mathbb{E}_{s' \sim p(s'|s,a)} [V^*(s')]$$

$$= \mathbb{E}_{s' \sim p(s'|s,a)} [r(s,a) + \gamma V^*(s')]$$

(Reward at t = 0) + gamma * (Return from expected state at t=1)

- Equations relating optimal quantities

$$V^*(s) = \max_a Q^*(s,a)$$

$$\pi^*(s) = \arg\max_a Q^*(s,a)$$

- Recursive Bellman optimality equation

$$\begin{aligned} Q^*(s,a) &= \mathop{\mathbb{E}}_{s' \sim p(s'|s,a)} \left[r(s,a) + \gamma V^*(s) \right] \\ &= \sum_{s'} p(s'|s,a) \left[r(s,a) + \gamma V^*(s) \right] \\ &= \sum_{s'} p(s'|s,a) \left[r(s,a) + \gamma \max_a Q^*(s',a') \right] \end{aligned}$$

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$$V^*(s) = \max_a \sum_{s'} p(s'|s,a) \left[r(s,a) + \gamma V^*(s') \right]$$

Bellman Optimality Equations

Algorithm: Value Iteration

- Initialize values of all states to arbitrary values, e.g., all 0's.
- While not converged:
  - For each state: $$V^{i+1}(s) \leftarrow \max_a \sum_{s'} p(s'|s,a) \left[r(s,a) + \gamma V^i(s')\right]$$
- Repeat until convergence (no change in values)

$$V^0 \rightarrow V^1 \rightarrow V^2 \rightarrow \quad \cdots \rightarrow V^i \rightarrow \ldots \quad \rightarrow V^*$$

Time complexity per iteration $O(|\mathcal{S}|^2|\mathcal{A}|)$

Value Iteration Update:

$$V^{i+1}(s) \leftarrow \max_a \sum_{s'} p(s'|s,a) \left[r(s,a) + \gamma V^i(s')\right]$$

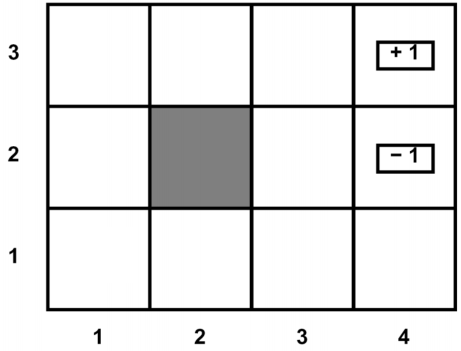
Q-Iteration Update:

$$Q^{i+1}(s,a) \leftarrow \sum_{s'} p\left(s'|s,a\right) \left[r\left(s,a\right) + \gamma \max_{a'} Q^i(s',a')\right]$$

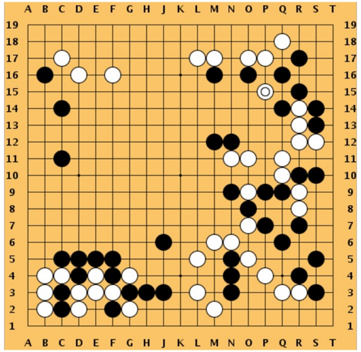
The algorithm is same as value iteration, but it loops over actions as well as states

Value iteration is almost never used in practice!

Time complexity per iteration $O(|\mathcal{S}|^2|\mathcal{A}|)$



$|S| = 11, |A| = 4$



$|S| \cong 3^{361}, |A| \cong 361$

$|S| \cong ?, |A| = ?$

Can't iterate over all (s, a) pairs -> need approximation!

We also don't know the transition function (model) -> need a *model-free* method!

Q-Learning

- We'd like to do Q-value updates to each Q-state:

$$Q'(s_t, a_t) \cong \sum_{s'} T(s_{t+1}|s_t, a_t)[r_t + \gamma \max_a Q(s_{t+1}, a)]$$

  - But can't compute this update without knowing all possible next states *s*'

- Instead, approximate the expectation with (lots of) experience samples
  - Take an action in the environment following *policy* $\text{argmax}_{\text{a}} Q(s, a)$
  - receive a sample transition (s_t, a_t, r_t, s_{t+1})
  - This sample suggests: $Q(s_t, a_t) \cong r_t + \gamma \max_a Q(s_{t+1}, a)$
  - Keep a running average to approximate the expectation:

$$Q'(s_t, a_t) = (1 - \alpha)Q(s_t, a_t) + \alpha[r_t + \gamma \max_a Q(s_{t+1}, a)]$$

Still need to represent all (s, a) pairs in a Q value table!

Q-Learning

Idea: represent the Q value table as a parametric function $Q_\theta(s, a)$!

How do we learn the function?

$$
\begin{aligned}
Q'(s_t, a_t) &= (1-\alpha)Q(s_t, a_t) + \alpha[r_t + \gamma \max_a Q(s_{t+1}, a)] \\
&= Q(s_t, a_t) + \alpha(r_t + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t))
\end{aligned}
$$

Now, at optimum, $Q(s_t, a_t) = Q'(s_t, a_t) = Q^*(s_t, a_t)$; This gives us:

$$0 = 0 + \alpha(r_t + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t))$$

Learning problem:

$$\text{argmin}_\theta || \underbrace{r_t + \gamma \max_a Q_\theta(s_{t+1}, a)}_{\text{Target Q value}} - Q_\theta(s_t, a_t)) ||$$

Deep Q-Learning

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- **Q-Learning** with **linear function approximators**

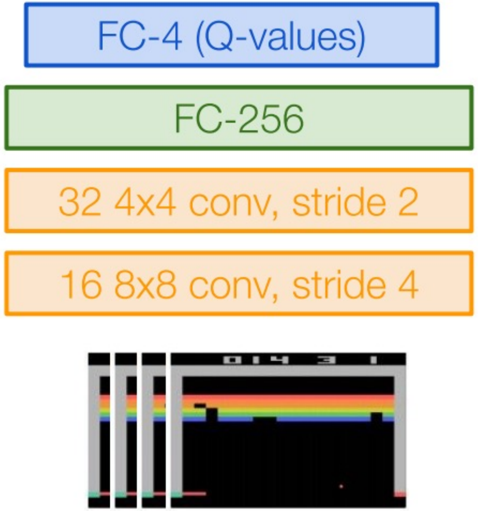
$$Q(s, a; w, b) = w_a^\top s + b_a$$

  - Has some theoretical guarantees

- **Deep Q-Learning**: Fit a **deep Q-Network**

$$Q(s, a; \theta)$$

  - Works well in practice
  - Q-Network can take arbitrary input (e.g. RGB images)
  - Assume discrete action space (e.g., left, right)



Deep Q-Learning

- Assume we have collected a dataset:

$$\{(s, a, s', r)_i\}_{i=1}^{N}$$

- We want a Q-function that satisfies bellman optimality (Q-value)

$$Q^*(s,a) = \mathop{\mathbb{E}}_{s' \sim p(s'|s,a)} \left[r(s,a) + \gamma \max_{a'} Q^*(s',a') \right]$$

- Loss for a single data point:

$$\text{MSE Loss} := \Big(\underbrace{Q_{new}(s,a)}_{\text{Predicted Q-Value}} - \underbrace{(r + \gamma \max_{a} Q_{old}(s',a))}_{\text{Target Q-Value}} \Big)^2$$

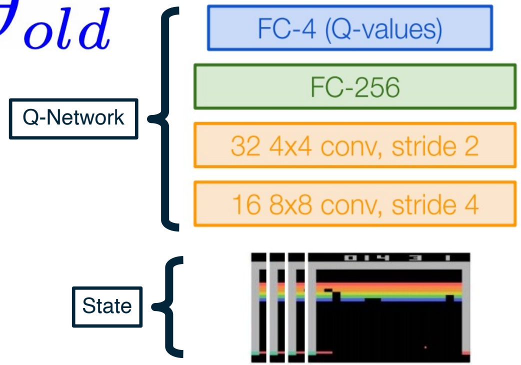
Deep Q-Learning

- Minibatch of $\{(s, a, s', r)_i\}_{i=1}^{B}$
- Forward pass: State → Q-Network → Q-Values per action

  $B \times D$ $\qquad$ $B \times n_{actions}$
- Compute loss:

$$\Big(\underbrace{Q_{new}(s,a)}_{\theta_{new}} - (r + \gamma \max_a \underbrace{Q_{old}(s',a)}_{\theta_{old}})\Big)^2$$

- Backward pass: $\dfrac{\partial Loss}{\partial \theta_{new}}$



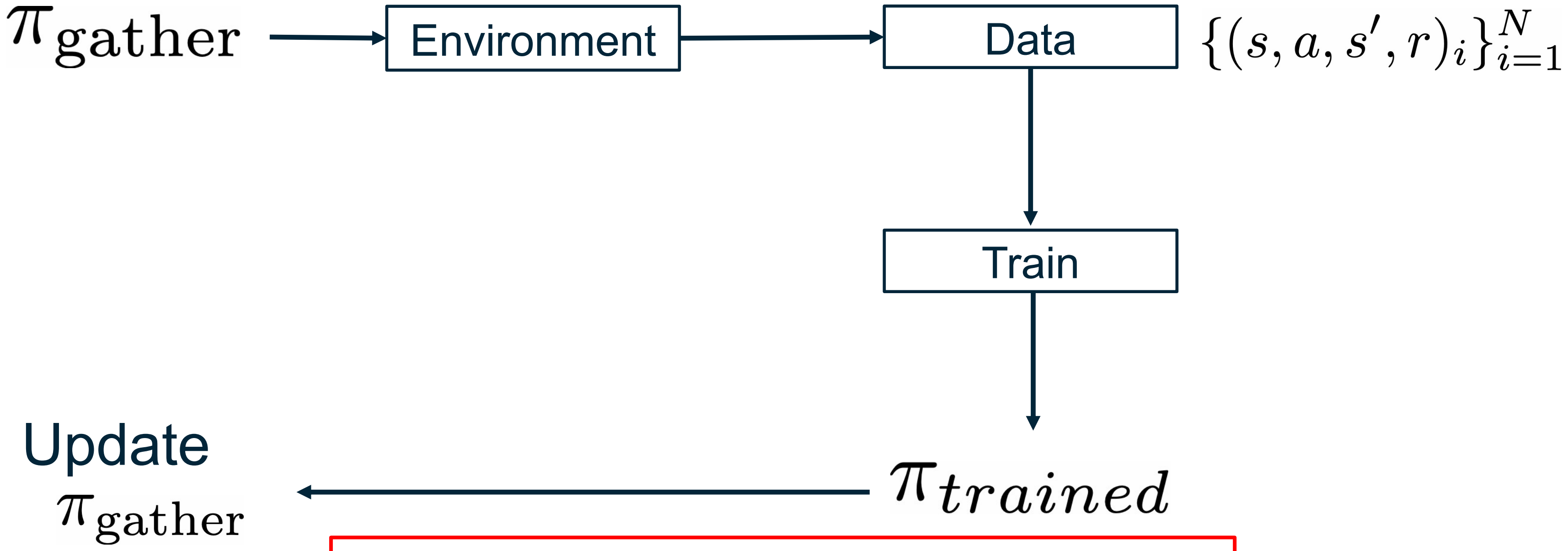
$$\text{MSE Loss} := \left(Q_{new}(s,a) - (r + \max_a Q_{old}(s',a))\right)^2$$

- In practice, for stability:
  - Freeze Q_{old} and update Q_{new} parameters
  - Set $Q_{old} \leftarrow Q_{new}$ at regular intervals or update as running average
    - $\theta_{old} = \beta\theta_{old} + (1-\beta)\theta_{new}$

How to gather experience?

$$\{(s, a, s', r)_i\}_{i=1}^{N}$$

This is why RL is hard



Challenge 1: Exploration vs Exploitation

Challenge 2: Non iid, highly correlated data

How to gather experience?

- What should π_{gather} be?
  - Greedy? -> no exploration, always choose the most confident action

$$\arg\max_a Q(s, a; \theta)$$

- An exploration strategy:
  - ϵ-greedy

$$a_t = \begin{cases} \arg\max_a Q(s, a) & \text{with probability } 1 - \epsilon \\ \text{random action} & \text{with probability } \epsilon \end{cases}$$

- Samples are correlated => high variance gradients => **inefficient learning**
- Current Q-network parameters determines next training samples => can lead to **bad feedback loops**
  - e.g. if maximizing action is to move right, training samples will be dominated by samples going right, may fall into local minima

Correlated Data Problem

- Correlated data: addressed by using experience replay
  - A replay buffer stores transitions (s, a, s', r)
  - Continually update replay buffer as game (experience) episodes are played, older samples discarded
  - Train Q-network on random minibatches of transitions from the replay memory, instead of consecutive samples
- Larger the buffer, lower the correlation

Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory $\mathcal{D}$ to capacity N
Initialize action-value function Q with random weights
for episode $= 1, M$ **do**
  Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
  for $t = 1, T$ **do**
    With probability ϵ select a random action a_t
    otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
    Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in $\mathcal{D}$
    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
    Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
  end for
end for

Experience Replay

Epsilon-greedy

Q Update

Deep Q-Learning Algorithm

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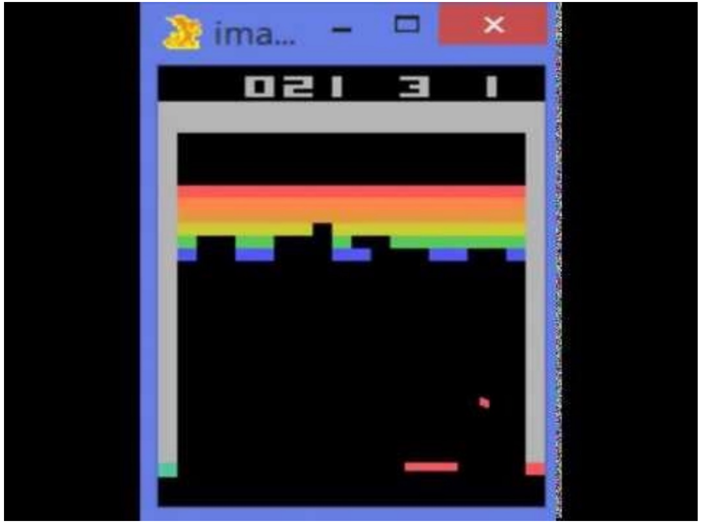
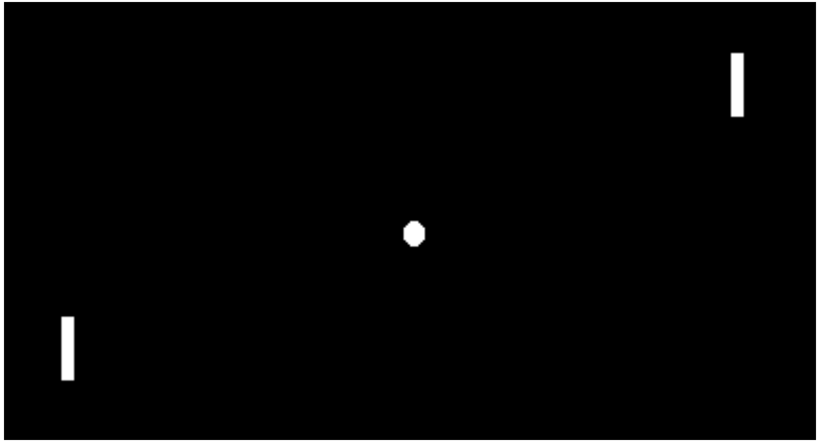
Atari Games

- **Objective**: Complete the game with the highest score
- **State**: Raw pixel inputs of the game state
- **Action**: Game controls e.g. Left, Right, Up, Down
- **Reward**: Score increase/decrease at each time step

Atari Games





https://www.youtube.com/watch?v=V1eYniJ0Rnk

Different RL Paradigms

- **Value-based RL**
  - (Deep) Q-Learning, approximating $Q^*(s, a)$ with a deep Q-network
- **Policy-based RL**
  - Directly approximate optimal policy π^* with a parametrized policy π_θ^*
- **Model-based RL**
  - Approximate transition function $T(s', a, s)$ and reward function $\mathcal{R}(s, a)$
  - Plan by looking ahead in the (approx.) future!

Today, we saw

- **MDPs**: Theoretical framework underlying RL, solving MDPs
- **Policy**: How an agents acts at states
- **Value function (Utility)**: How good is a particular state or state-action pair?

- **Solving an MDP with known rewards/transition**
  - **Value Iteration:** Bellman update to state value estimates
  - **Q-Value Iteration:** Bellman update to (state, action) value estimates

- **Policy Iteration**
  - Policy evaluation + refinement

Summary: MDP Algorithms

Next Time: RL continued --- Policy Gradient and Actor-Critic