# Skyline Graph

This note is about analyzing "Grid of tries"(Chapter 12.5.3 in the textbook) with skyline graph.

## Definitions

**dst_len, src_len**: length of the destination prefix or the source prefix

**Skyline graph**: A graph with dst_len and src_len as vertical and horizontal axis. Given a pair of (dst ip, src ip), all and only the matching nodes (including those have exact-matching rules or not) will be on the graph. A node's coordination is (src_len , dst_len). Every horizontal line in the skyline graph represents a src trie.

**Domination rule**: In a skyline graph, if a rule is not a prefix in both dst and src fields of any other rule, then the rule is a domination rule.

**Domination node**: In a skyline graph, if a node has the longest dst_len among all nodes with a particular src_len , then the node is a domination node.

Property of the domination node: A domination node cannot be a prefix in both dst and src fields of any other node in a skyline graph.

To prove it, you can assume a domination node is at (s1 , d1), and there exists a node at (s2 , d2), s2 > s1 , d2 > d1.

If there is a node is at (s2 , d2), there must be nodes at (0 , d2) , (1 , d2) , ... , (s1 , d2) , ... , (s2 - 1 , d2), because a node must be on a src trie.

The node at (s1 , d2) now should be the domination node, which is a contradiction.

**Skyline**: the algorithm's search path in the src tries

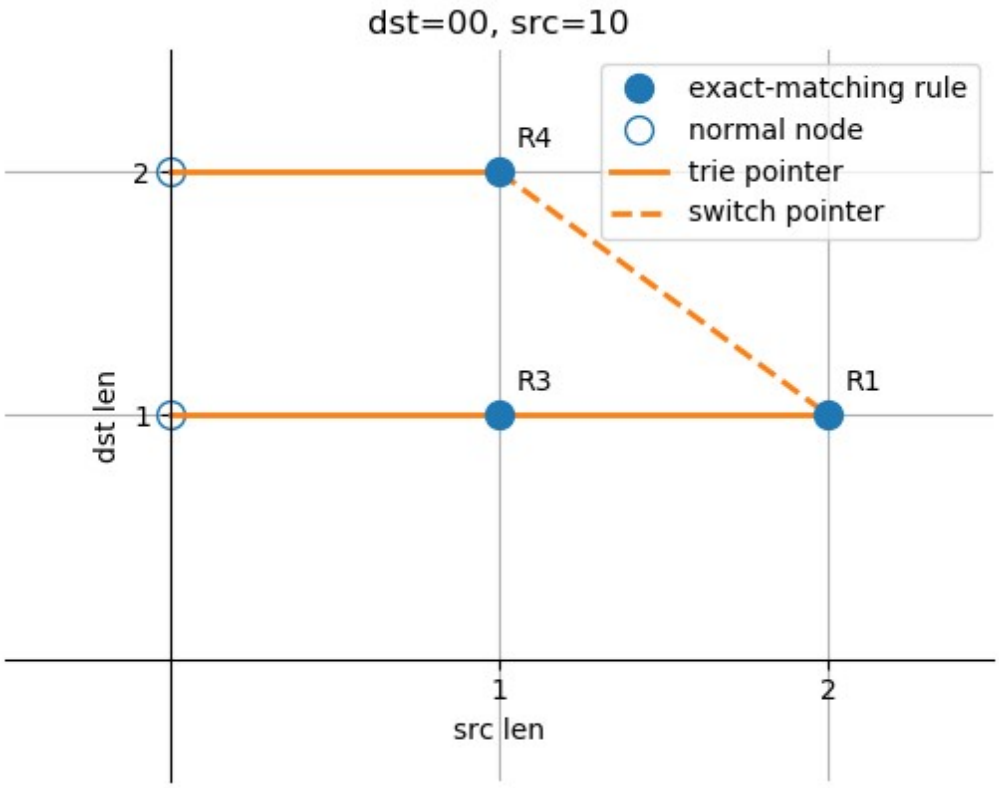
Some properties of the skyline:

- The first node in the skyline always has src_len = 0. Because you always search from the root of a src trie.
- The first node in the skyline always has the longest possible dst_len. Because the algorithm first go to the deepest possible dst trie node.
- dst_len is non-increasing. Because:

    - If next node is on the same trie, dst_len remains the same.
    - If following the switch pointer, you will switch to a src trie corresponding to an ancestor dst trie node, so dst_len must decrease.
- src_len will increase by 1 every step of the skyline. Because to go along a switch pointer or to find the child both will go 1 level down the src trie.

### Example

Using the example in the book, a skyline graph looks like this:




## Claim 1

> It is impossible that a skyline miss a domination node in a skyline graph.

The first node of the skyline must be a domination node, because the algorithm will start at the src trie with the longest dst_len.

Then, by definition, every step, the algorithm first attempts to search its child. If its child is a matching node, it must be a domination node, because it has the longest possible dst_len for a particular src_len. If its child is not a matching node, the algorithm will use a switch pointer to find the lowest ancestor, and the lowest ancestor has the longest dst_len for a

particular src_len, so it is also the domination node.

## Claim 2

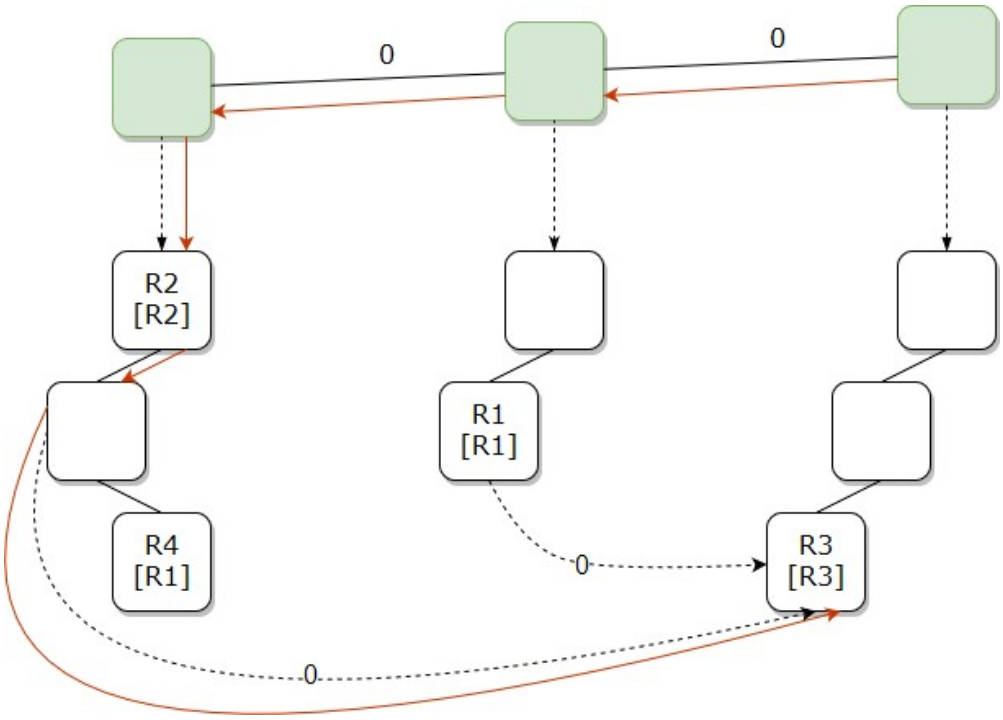
> It is possible that a skyline miss a domination rule in a skyline graph.

### Example

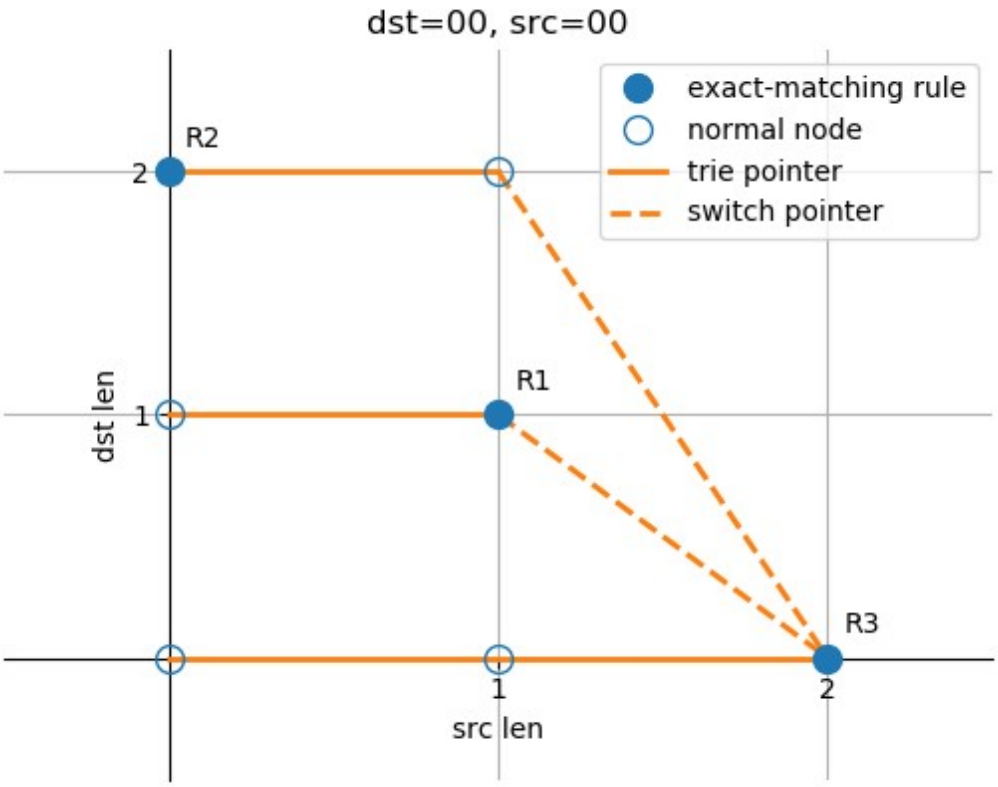
| Rules | Dst | Src |
|---|---|---|
| R1 | 0* | 0* |
| R2 | 00* | * |
| R3 | * | 00* |
| R4 | 00* | 01* |

In this rule set, priority is defined as R1 > R2 > R3 > R4.

The above rule set will produce a grid-trie (rules in "[ ]" are *storedRules*):

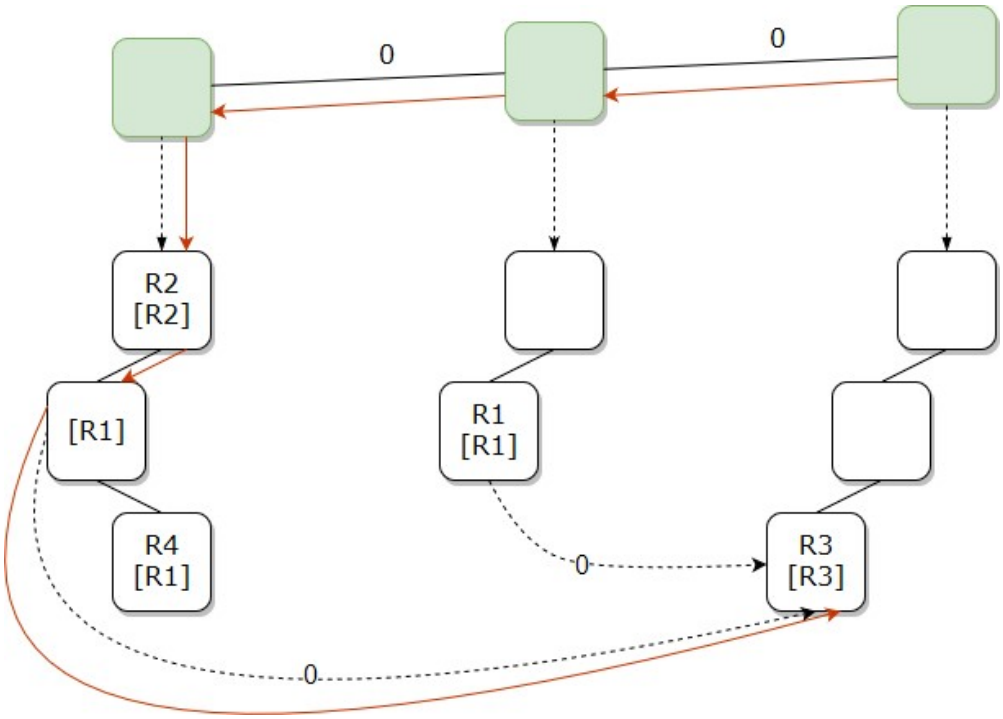



If dst = 00, src = 00, then the skyline graph will look like this:




As shown above, R1 cannot be reached, but R1 is a domination rule. If a *storedRule* is only stored in every exact-matching rule node, the algorithm will eventually output R2 instead of the correct answer R1.

If a *storedRule* is stored in every node, the algorithm is correct:



## Claim 3

> The algorithm can cover all the matching rules if a *storedRule* is stored in every node.

Suppose an exact-matching rule node's src_len is s1. The domination node with src_len = s1 will consider this rule. Since a skyline does not miss any domination node, the algorithm can cover all the matching rules.