Homework 7.5
Bayesian Networks

In your friendly neighborhood nuclear power plant, there is an alarm that senses whether a particular temperature gauge reports a temperature that is too high. The gauge itself reads either "high" or "normal". The recently-laid-off technician estimates that on any given day the actual temperature is too high about 2% of the time.

Please work with the following *Boolean* variables:

A	Alarm sounds
FA	Alarm is faulty
G	Gauge reports high temperature
FG	Gauge is faulty
T	Actual temperature is too high

Complete the following problems about this power plant:

A. Draw a Bayesian network for this problem domain.

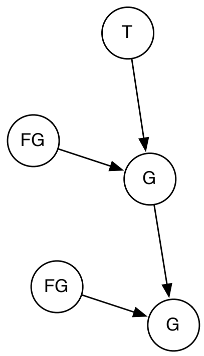
B. Suppose that the probability that the gauge reports the temperature accurately is *x* when it is working, and *y* when it is faulty. Write the conditional probability table for G, conditioned on all of its parent(s) in your Bayes net.

C. Suppose that the alarm works correctly at all times except when it is faulty, in which case it never sounds. Write the conditional probability table for A, when conditioned on all of its parent(s) in your Bayes net.

D. Suppose we know the alarm *and* gauge are working properly, and the alarm sounds! Write an expression for the probability that the actual temperature is too high. Please show your steps. (Hint #1: What do you know about G given that the alarm is sounding and not faulty?) (Hint #2: Is there an opportunity to write a marginal?)

Solutions:

A.



B.

	fg		**~fg**	
	t	**~t**	**t**	**~t**
g	*y*	1 - *y*	*x*	1 - *x*
~g	1 - *y*	*y*	1 - *x*	*x*

C.

	fa		**~fa**	
	g	**~g**	**g**	**~g**
a	0	0	1	0
~a	1	1	0	1

Answer to D. on following page…

D.

We know $FA = false$, $FG = false$, $A = true$. Because the conditional $P(A|FA, G)$ is all-or-nothing, and the gauge is working and the alarm is sounding, we know that $G = true$ as well. We have 4 observed variables, and we want to infer the probability of T, whether the temperature is too high. That means that we need to know $P(T|FA, FG, A, G)$. Using Bayes' law, we can write this as

$$P(T|FA, FG, A, G) = \frac{P(FG, FA, A, G|T)\, P(T)}{P(FG, FA, A, G)} \tag{1}$$

Now we need to expand these terms so that we can read them off of the known distributions. The first term in the numerator, $P(FG, FA, A, G|T)$ can be split up according to the rules of conditional independence in Bayesian networks:

$$P(FG, FA, A, G|T) = P(A|FA, G)\, P(G|FG, T)\, P(FA)\, P(FG) \tag{2}$$

The denominator is only one variable away from being the joint! We can simply write it as a marginal:

$$P(FG, FA, A, G) = \sum_T P(A|FA, G)\, P(G|FG, T)\, P(FA)\, P(FG)\, P(T) \tag{3}$$

Now we can re-write the conditional we are interested in from Eq 1:

$$P(T|FA, FG, A, G) = \frac{P(A|FA, G)\, P(G|FG, T)\, P(FA)\, P(FG)\, P(T)}{\sum_T P(A|FA, G)\, P(G|FG, T)\, P(FA)\, P(FG)\, P(T)} \tag{4}$$

When we pull the terms in the denominator that do not depend on T out of the sum over T, many terms in the numerator and denominator cancel:

$$P(T|FA, FG, A, G) = \frac{P(G|FG, T)\, P(T)}{\sum_T P(G|FG, T)\, P(T)} \tag{5}$$

We are almost done. We need to evaluate the sum in the denominator, which is easy because we already have values for the variables, and we know the distributions:

$$\begin{aligned} \sum_T P(G|FG, T)\, P(T) &= P(g|\neg fg, \neg t)\, P(\neg t) + P(g|\neg fg, t)\, P(t) \\ &= (1 - x)(0.98) + (x)(0.02) \end{aligned} \tag{6}$$

Now we get the actual conditional distribution over T by simply evaluating our expression for $P(T|FA, FG, A, G)$ in Eq. 5:

$$P(T|FA, FG, A, G) = \begin{cases} \dfrac{(1 - x)(0.98)}{(1 - x)(0.98) + (x)(0.02)}, & \neg t \\ \dfrac{(x)(0.02)}{(1 - x)(0.98) + (x)(0.02)}, & t \end{cases} \tag{7}$$