CS4600 - Introduction to Intelligent Systems
Fall 2003

Homework 7 - Sample Solution

Problem 1

Of the entire population, 2% has a certain disease X. A test Y, which indicates whether or not a person has the disease, is not 100% accurate. If a person has the disease, there is a 6% chance that it will go undetected by the test. However, there is also a 9% chance of "false alarm" (meaning that the person does not have the disease but the test indicates otherwise). A person Z takes a test which later comes out positive (meaning that the test says he has the disease). What is the probability of this person having the disease in reality?

Let D be"having the disease"
+ be "test positive"

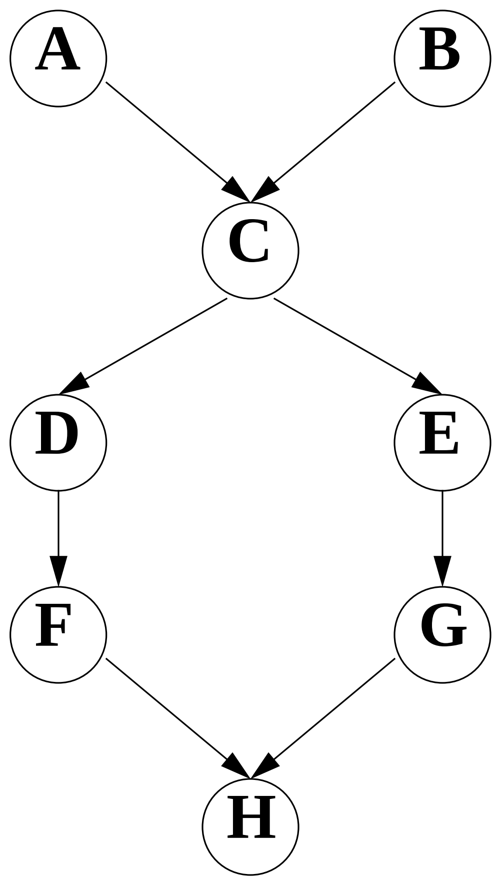
We are given the following information:
P(D) = 0.02
which implies P(not D) = 0.98
P(not + | D) = 0.06
which implies P(+ | D) = 0.94
P(+ | not D) = 0.09

First, we compute P(+)
= P(+ AND D) + P(+ AND (not D))
= P(+ | D) P(D) + P(+ | not D) P(not D)
= 0.94 x 0.02 + 0.09 x 0.98
= 0.107

We would like to know P(D | +)
= P(+ | D) x P(D) / P(+)
= 0.94 x 0.02 / 0.107
~= 0.1757

Problem 2

Consider the following Bayesian network:



a) Are D and E necessarily independent given evidence about both A and B?

No. The path D-C-E is not blocked.

b) Are A and C necessarily independent given evidence about D?

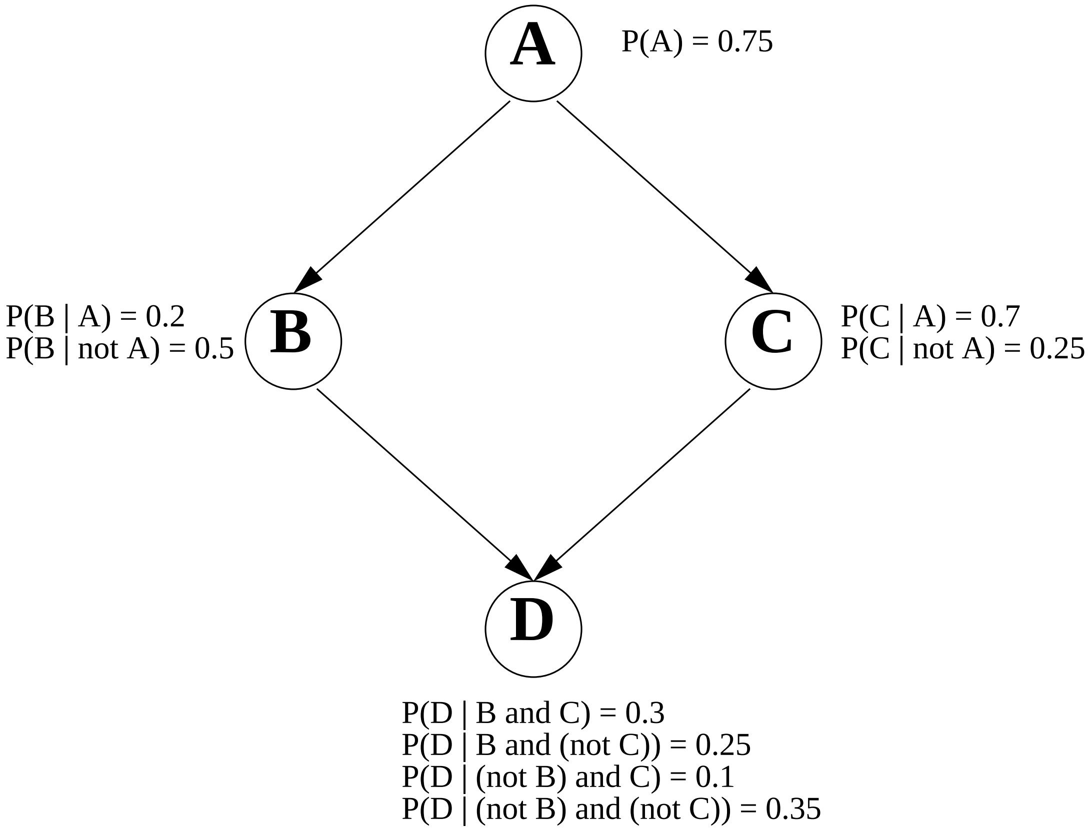
No. They are directly dependent. The path A-C is not blocked.

c) Are A and H necessarily independent given evidence about C?

Yes. All paths from A to H are blocked.

Problem 3

Consider the following Bayesian network. A, B, C, and D each could have a value of either true or false. If we know that A is true, what is the probability of D being true?



P(D | A)

$$= \sum_{(b,c) \in B \times C} P(D \mid (B,C) = (b,c)) \times P((B,C) = (b,c) \mid A)$$

= P(D | B and C) x P(B and C | A) +
P(D | B and (not C)) x P(B and (not C) | A) +
P(D | (not B) and C) x P((not B) and C | A) +
P(D | (not B) and (not C)) x P((not B) and (not C) | A)

= (0.3 x 0.2 x 0.7) + (0.25 x 0.2 x 0.3) + (0.1 x 0.8 x 0.7) + (0.35 x 0.8 x 0.3)

= 0.042 + 0.015 + 0.056 + 0.084

= 0.197