The problem:

1: The customs officials search everyone who entered the country who was not a VIP
2: Some drug pushers entered the country and they were only searched by drug pushers
3: No drug pusher was a VIP

Conclusion: Some of the officials were drug pushers?

Predicates:

- E(x) – x entered the country
- S(x,y) – x is searched by y
- C(x) – x is a customs official
- V(x) – x is a VIP
- P(x) – x is a drug pusher

Convert each sentence into first-order logic:

1: $\forall x\ [\ (\ E(x) \wedge \neg V(x)\ ) \rightarrow (\exists y\ (\ S(x,y) \wedge C(y))\ )\ ]$

Eliminate implication:
$\forall x\ [\ \neg(E(x) \wedge \neg V(x)) \vee (\exists y\ (\ S(x,y) \wedge C(y))\ )\ ]$

Move nots inward:
$\forall x\ [\ \ (\ \neg E(x) \vee V(x)\ ) \vee (\exists y\ (\ S(x,y) \wedge C(y))\ )\ ]$

Skolemize:
$\forall x\ [\ (\ \neg E(x) \vee V(x)\ ) \vee (\ S(x,f(x)) \wedge C(f(x))\ )\ ]$
Skolemization is the process of eliminating existential quantifications by replacing each instance of y with a function of x that selects the thing that the existential would have selected. This function is called a <u>skolem function</u>. The reason why the skolem function takes x as a parameter is because it is within the scope of the universal quantifier referring to x. The choice of y is dependent on what x is.

Drop the universal:
$(\ \neg E(x) \vee V(x)\ ) \vee (\ S(x,f(x)) \wedge C(f(x))\ )$
At this point, we can simply assume that any variable that has survived is universally quantified.

Distribute ors over ands:
$(\ \neg E(x) \vee V(x) \vee S(x,f(x))\ ) \wedge (\ \neg E(x) \vee V(x) \vee C(f(x))\ )$

2: $\exists x\ [\ P(x) \wedge E(x) \wedge \forall y\ (S(x,y) \rightarrow P(y)\ )\ ]$

Eliminate implications:
$\exists x\ [\ P(x) \land E(x) \land \forall y\ (\neg S(x,y) \lor P(y)\ )\ ]$

Skolemize:
$P(a) \land E(a) \land \forall y\ (\neg S(a,y) \lor P(y)\ )$
Skolemization replaces the variable x with a symbol *a* that is the thing that would be selected by the existential quantifier. That is, we named the thing selected by the quantifier as the skolem constant *a*. This is equivalent to using a skolem function with no parametesr, f(). The reason why the skolem function has no parameters is becaue the existential quantifier is not within the scope of any other quantifiers.

Drop the universal:
$P(a) \land E(a) \land (\neg S(a,y) \lor P(y)\ )$

3: $\forall x\ [\ P(x) \rightarrow \neg V(x)\ ]$

Eliminate implications:
$\forall x\ [\ \neg P(x) \lor \neg V(x)\ ]$

Drop universal:
$\neg P(x) \lor \neg V(x)$

4: $\neg\exists x\ [\ C(x) \land P(x)\ ]$

Move nots inward:
$\forall x\ [\neg C(x) \lor \neg P(x)\ ]$

Drop universal:
$\neg C(x) \lor \neg P(x)$

Resolution:

1a: ¬E(x) ∨ V(x) ∨ S(x,f(x))
1b: ¬E(x) ∨ V(x) ∨ C(f(x))
2a: P(a)
2b: E(a)
2c: ¬S(a,y) ∨ P(y)
3: ¬P(x) ∨ ¬V(x)
4: ¬C(x) ∨ ¬P(x)